A STRATEGY-BASED TRANSIT ASSIGNMENT MODEL FOR UNRELIABLE NETWORKS

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1 ABSTRACT

In this paper we present a model how passengers navigate through overcrowded public transit networks. The model is then integrated into a transit assignment procedure. Variations of this model can be used to reflect different levels of information. This allows estimating the benefit a portable journey planner might have for passengers who need to navigate through unreliable networks.

In many cities public transportation systems have reached their capacity limit. During the peak hours vehicles are crowded and often passengers are left behind on the platform. In some cases subway stations need to be temporarily closed in order to prevent overcrowding on the platforms. Passengers are aware of these facts and take them into account when they choose their route and their departure time.

When a public transit network reaches its capacity limit it is no longer reliable from the point of view of the passenger. It no longer suffices to choose a path through the network, since it may contain a ride on a vehicle that might be overcrowded. Instead, a passenger needs to have a strategy to navigate through such an unreliable network. He has to determine what to do, if he fails to board a vehicle.

A schedule-based transit assignment model that is based on strategies was presented by Hamdouch and Lawphongpanich (2008). In that model a strategy consists of an ordered set of choices at each station. When the capacity of a vehicle is too small to accommodate all passengers, a passenger who is unable to board the first vehicle of his choice tries to board the second best and so forth. An optimal strategy minimizes the expected generalized cost of travel. The expected cost depends on the cost of the possible outcomes and their probability. As opposed to a single path, a strategy is always reliable.

In this paper we analyze two use cases for this strategy concept. The first use case is a portable journey planner that guides a passenger through an unreliable network. The input is information about the schedule and the crowding situation inside vehicles and at stations. We analyze to what extent additional information improves the resulting strategy compared to a passenger who has less information or other restrictions. Passengers may be unaware of the capacity restrictions or have less knowledge of the schedule and navigate based on headways; or – due to the complexity of the network – they restrict their choices to certain connections that are deemed attractive.

The results may be used as an indicator how portable journey planners that access online information about the crowding situation can improve the travel times of the passengers.
The second use case is transit assignment. We present a transit assignment model that is based on strategies. It seems plausible that in unreliable networks passengers leave earlier than their desired departure time in order to account for the unreliability. We show that this effect can be reproduced in our model.

2 INTRODUCTION

The purpose of transit assignment is to determine how passengers navigate through a public transit network. In static models, demand is expressed as an OD matrix, which contains a number of passengers who want to travel from an origin O to a destination D. The network consists of a set of stops and lines, which connect the stops. Since the model is static, no departure times are given for the lines. The passengers rather navigate based on average line headways. It is assumed that vehicles arrive in random order, and that passengers board vehicles they deem attractive. Therefore, it is not known a priori, which path through the network a single passenger will take. In the assignment, the OD flows are split according to the arrival probabilities of the lines. The approach has become known as the *hyperpath approach*, or alternatively as *optimal strategy approach* (Nguyen & Pallottino, 1988; Spiess & Florian, 1989).
Due to the static nature of these models, it is not easy to model peak hour effects. While there are approaches that can model the effect of congestion on route choice (this is done by reducing the frequencies of congested lines, the *effective frequency approach (Cominetti & Correa, 2001)*), it seems to be hard to model the effect on departure time choice.
In dynamic transit assignment models, a schedule is given. Therefore, these models are usually called schedule-based models. The demand has a temporal dimension, which allows modelling peak hours and off-peak hours. The network and the schedule are usually represented with a time-expanded network. The details of the time-expanded models tend to be different, while the general structure tends to be the same. It is possible to develop transit assignment models that are based on shortest paths in the time-expanded network. However, usually other approaches have been preferred. The majority of the schedule-based assignment models are more complex. Friedrich and Wekeck (2004) presented a model based on a branch and bound search algorithm, that allows modelling costs that depend on path legs, not only on individual links of the path. Nuzzolo et al. (Nuzzolo, Crisalli & Rosati, 2011; Russo, 2004) developed a complex model that is based on a so-called *diachronic network*. The passengers' choices are based on a random utility model. Furthermore, it is assumed that passengers navigate based on frequencies in high-frequency networks, not on the schedule. Recently, this model was extended, so that it is now possible to take into account vehicle capacity constraints, which may not be violated. Hamdouch et al. (2008) presented a model, where vehicles have fixed capacity constraints.

Passengers are aware of these constraints. Instead of paths, they choose strategies, which take into account the possibility of failure to board a vehicle. This means that the passengers do not simply react, when they fail to board a vehicle; instead, they make a pre-trip choice that takes into account all the failure probabilities in the network.
Our model is based on the model from Hamdouch et al. In that model it is assumed that passengers know the complete network and its schedule. Furthermore, it is assumed that passengers have precise knowledge of the failure-to-board probabilities in the whole network. We will show that it is also possible to model passengers, who make their choices based on average line headways and less precise knowledge of the failure-to-board probabilities. This is supposed to be a more precise method of modelling passenger behaviour in high-frequency networks. The full-information and the headway-based approaches are then compared.
If one assumes that passengers are unable to know the complete schedule and the reliabilities, one might argue that the full-information strategy could still be calculated by a portable navigation device. This device could, for example, have access to a server, which has information about the schedule and how congested the vehicles are. The headway-based approach represents a well-informed (but not completely informed) passenger. The difference in the quality of the strategies can be seen as the potential benefit of a portable navigation device.

3 NETWORK MODEL

The public transit network and timetable are represented by a time-expanded network $G = (N, A)$, where N is the set of nodes and A is the set of arcs. There are two types of nodes: the set of *stop nodes* $N_T \subset N$ and the set of *in-vehicle nodes* $N_V \subset N$. Stop nodes represent being at a stop at a certain point in time $\tau(n)$, in-vehicle nodes represent being inside a vehicle between two stops. There are five types of arcs. *Waiting arcs* connect consecutive stop nodes at the same stop. *Dwelling arcs* connect consecutive in-vehicle nodes of a vehicle run. *Boarding arcs* start at a stop node and end at an in-vehicle node. *Alighting arcs* start at an in-vehicle node and end at a stop node. An in-vehicle node has exactly one incident boarding arc and one incident alighting arc. The fifth type of arc is the *walking arc*. As waiting arcs they connect stop nodes, but of different stops. They represent walking from one stop to the other.
Figure 1 shows a small time-expanded network with four stops. The x-axis represents space, the y-axis represents time.

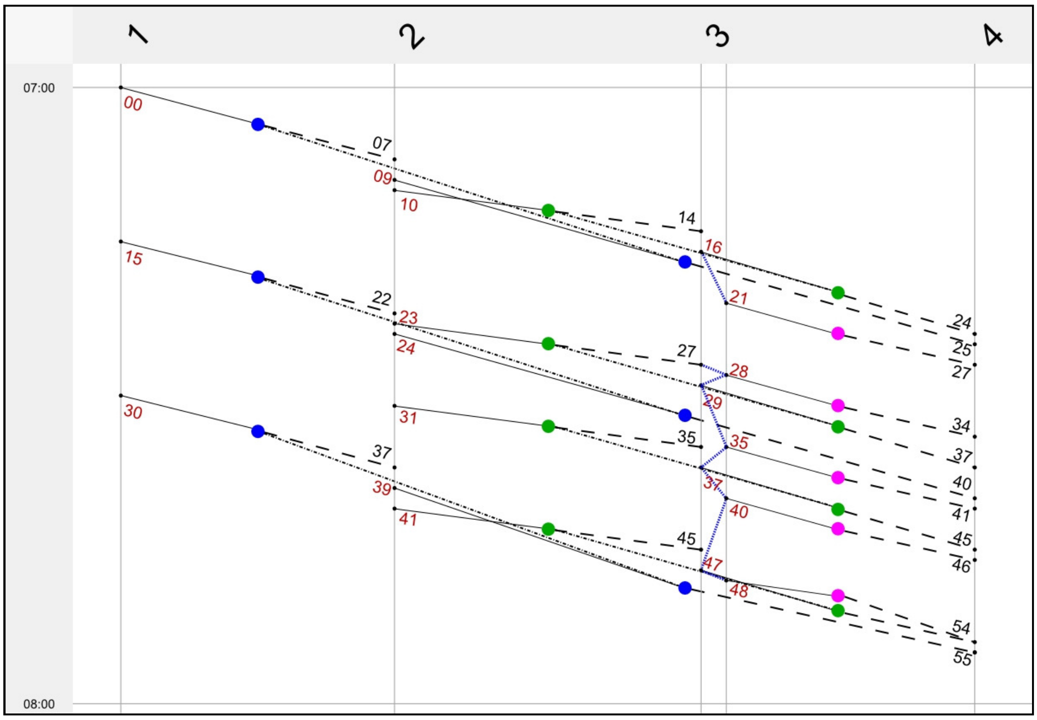

Figure 1: A Small Time-Expanded Network

The node, where an arc a starts, is also called the *tail node* $t(a)$; the node, where an arc ends, is called *head node* $h(a)$. Stop nodes have a time coordinate $\tau(n)$. For stop nodes this is the point in time which they represent.

Nodes and arcs have intrinsic costs. For an in-vehicle node $n \in N_V$, the intrinsic cost $b(n)$ depends on the length of the journey that the in-vehicle node represents. Suppose that the corresponding boarding arc is called a and the corresponding alighting arc is called a'. Then

$$b(n) := \tau\big(h(a')\big) - \tau\big(t(a)\big), n \in N_V.$$

For stop nodes, the intrinsic cost is 0. The intrinsic cost c of an arc a depends on the type of arc and the time coordinate of the end nodes. If $a \in A_W$ is a waiting arc (where A_W is the set of waiting arcs), the cost is

$$c(a) := \beta_1 \Big(\tau\big(h(a)\big) - \tau\big(t(a)\big)\Big), a \in A_W.$$

Here, β_1 is a positive cost parameter that sets the cost of waiting at a stop in relation to the cost of staying in a vehicle. Usually it is assumed that passengers prefer being inside a vehicle to waiting at a stop, therefore it is assumed that $\beta_1 \geq 1$. The intrinsic cost for walking arcs $a \in A_L$, where A_L is the set of walking arcs, is defined similarly:

$$c(a) := \beta_2 \Big(\tau\big(h(a)\big) - \tau\big(t(a)\big)\Big), a \in A_L.$$

Here, β_2 is the parameter that sets walking into relation with staying in a vehicle. The cost of boarding arcs is 0. The intrinsic cost of alighting arcs can be used to model the passengers' averseness to transfers; therefore, a penalty ρ can be used if the alighting arc does not end at the destination of the passenger. It is assumed that dwelling inside a vehicle as it is waiting at a stop has the same cost as being inside the vehicle when it is moving.

In our model, vehicles have capacity constraints. These constraints may not be violated. This means that if a vehicle is full, the passengers remaining at the stop may not enter it; they have to wait for the next vehicle or take a different route to their destination. A vehicle at a certain stop therefore has a certain access probability, or *reliability* r. The reliability function r is defined on the set of arcs A and takes values between 0 and 1. Its value is 1 for waiting, walking, dwelling and alighting arcs. For boarding arcs it can be smaller than 1. The reliability for boarding arcs is high, if there are not many passengers. This is usually true in the normal, off-peak times. At peak times the reliability of boarding arcs is low, if vehicle capacities are insufficient.

3.1 Strategies

When passengers navigate through the network, they know that vehicles may have insufficient capacity. The passengers take this into account before they start their journey. This means that a passenger has one or more alternatives for the case that he fails to board a vehicle. These alternatives are called *options*. They coincide with the outbound arcs of the node. At each node n, a passenger has a sorted set of options $\sigma(n) = \{a_1, \dots, a_j\}$. When the first option fails, a passenger tries to use the second option, and so forth. It is assumed that the set of options contains at least one reliable arc. Usually at stop nodes this is the waiting arc.

All passengers have an origin, a destination, and a strategy to travel from the origin to the destination. For this, a passenger selects a node from the set of nodes in his origin and navigates through the network according to the sorting of options at each node that he encounters. The passenger leaves the network at the first node he reaches that belongs to his destination. It is assumed that there always exist reliable ways to travel from the origin to the destination, so no passengers are “lost” in the network.

The passenger selects the node at the origin (the *root* node n_r) by its cost $b(n_r)$. One way to define the cost of a node is the *mean value cost function* that was used in (Hamdouch et al., 2008). Here the cost c' of each option a_i is multiplied with its probability π, and then the costs of the options are summed up. The cost c' of each option is derived from the intrinsic cost c of its arc and the cost b of the end node of that arc:

$$c'(a_i) = c(a_i) + b(h(a_i))$$

$$\pi(a_i) \coloneqq r(a_i) \cdot \prod_{k=1}^{i-1} 1 - r(a_k)$$

$$b(n) \coloneqq \sum_{i=1}^{j} \pi(a_i) c'(a_i)$$

A passenger can have a desired arrival time or a desired departure time. In this paper we focus on passengers with a desired departure time. A passenger will select a root node n_r, where the sum of the node cost $b(n_r)$ and the schedule delay penalty $SD(n_r)$ are minimal. The schedule delay penalty SD is expressed with the following schedule delay function:

$$SD(n_r) \coloneqq \gamma_1 \cdot \max(0, \tau(P) - \tau(n_r)) + \gamma_2 \cdot \max\big(0, \tau(n_r) - \tau(P)\big)$$

Schedule delay functions of this type are used frequently in the literature. In many cases they have an additional one-time penalty term (Noland & Polak, 2002). Here, $\tau(P)$ is the desired departure time of the passenger, and γ_1 and γ_2 are non-negative parameters.

3.2 Information

When the mean value cost function is used, it is assumed that the passenger has complete information about the network. This includes knowledge of the schedule and knowledge about the reliability of each connection. When frequencies are high, though, knowledge of individual vehicle departure times has a lower value than in low-frequency networks – passengers then navigate based on average headways (Nuzzolo, Russo & Crisalli, 2003).

In headway-based models, choices are made based on line properties, not on properties of individual vehicles. A passenger determines an attractive set of lines and boards the first vehicle of an attractive line that arrives. A commonly used model to determine the attractive set of lines uses the following formula (Chriqui & Robillard, 1975):

$$\min_X \frac{1 + \sum \lambda_l \overline{c_l} x_l}{\sum \lambda_l\, x_l}$$

Here λ_l is the frequency of line l, $\overline{c_l}$ is its cost, and x_l is a 0/1 variable determining whether line l belongs to the attractive set of lines. When the expression in the formula above is minimized, only the attractive lines have the value $x_l = 1$. Under the assumption that headways are exponentially and independently distributed, it can be shown that a simple greedy method suffices to optimize the above expression (Spiess et al., 1989).

In order to use this approach in the schedule-based strategy model, the values λ_l and $\overline{c_l}$ have to be defined for each line. In the following, ways are proposed how this can be done. These models still have to be confirmed with actual data. The preliminary results that are presented in the example section are based on schematic network models. They seem to confirm that the ideas point in the right direction.

Suppose a passenger P is standing at a stop at time τ. Let l be a line, then $\tau_{l,1}, \tau_{l,2}, \ldots$ are the next departure times of that line. Suppose that $c'_{l,1}, c'_{l,2}, \ldots$ are the costs of the line departures and $r_{l,1}, r_{l,2}, \ldots$ their reliabilities.

The first modelling decision is to define the set of lines on a stop. This is simple in static models. In a dynamic model, the problem is that lines may only run early in the morning or late at night. This means that the set of lines itself depends on the time-of-day. Furthermore, there may be lines with low frequencies, for example one vehicle per hour, even if the network has many high-frequency lines. It seems that if the departure time of such a vehicle is near, it will affect the choices of the passengers; if the vehicle has just departed, it is likely that the line plays no role in the passengers' choices. A possible way to determine the set of lines is to take only lines into account that have a vehicle departure in the next 30 minutes. Therefore, a line l enters the set of lines passenger P perceives, if

$$\tau_{l,1} - \tau \leq 30 \text{ min.}$$

The next modelling decision is how to define the cost of a line. Again, this is more complex than in the static case, because there the cost is constant; it does not depend on the individual vehicle. In the dynamic case, though, the attractiveness of a line's destination may change. For example, if it is a bus line that leads to a subway station of a central line, it may be attractive in off-peak hours, but become unattractive in peak hours, because the trains are overcrowded. One possibility to define the cost of a line is to take the average cost of the next two vehicle departures of that line:

$$\overline{c_l} := \frac{c'_{l,1} + c'_{l,2}}{2}$$

The next modelling decision is how to define the line headways. One way to do this is to take the difference between the next two departures of the line. A preliminary definition for the line frequency then would be

$$\lambda'_l := \frac{1}{\tau_{l,2} - \tau_{l,2}}$$

However, the passengers should also take into account that the vehicles could be unreliable. The reliability of the line may, again, be defined by the average reliability of the next two vehicle runs:

$$r_l := \frac{r_{l,1} + r_{l,2}}{2}$$

Assuming that the headways are distributed with an exponential distribution, the actual line frequency can be adjusted (Trozzi, Hosseinloo, Gentile & Bell, 2009):

$$\lambda_l := \frac{1}{\tau_{l,2} - \tau_{l,2}} \cdot r$$

It is important to notice that due to the changes in costs and frequencies, the set of attractive lines changes during the day. Therefore, it can happen that passengers change their opinion about the attractiveness of lines while they are waiting at a stop. A vehicle of a line may be perceived attractive even if a passenger let a vehicle of the same line pass ten minutes ago.

4 ASSIGNMENT

The assignment procedure is an iterative process. At first, optimal strategies are determined for all origins and destinations. Then the demand is loaded onto the network. For this, a packet-based procedure is used (see also Hamdouch et al., 2008; Nuzzolo et al., 2011). During the loading, a boarding process is performed on a stop at each point in time when a vehicle leaves. The boarding process provides new reliabilities of the boarding arcs. It is assumed that the boarding process is random in our examples. Another possibility is first-in-first-out boarding. The resulting reliabilities are used in the next iteration of the algorithm to calculate new optimal strategies.
Figure 2 shows a flow chart of the assignment procedure.

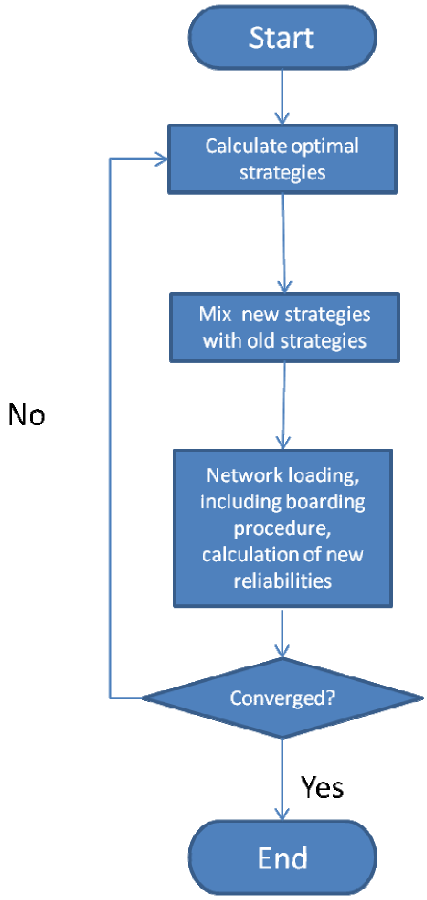

Figure 2: The Assignment Procedure

5 EXAMPLES

The following examples show very simple networks. Despite their simplicity, the assignment results and the strategy choices are not trivial. The parameters of the examples were chosen in such a way that the results are better tractable computationally. All the parameters (are assumed to be 1.0, which means that waiting has the same cost as travelling inside a vehicle and has the same cost as moving the departure time backward or forward.

5.1 One Line with Capacity Constraints

In Example 1 the network has two stops and one line. The line takes 10 minutes to travel from Stop 1 to Stop 2. It runs every 4 minutes, starting at 6:00 and ending at 7:56. Each vehicle has a capacity of 100 passengers. Figure 3 shows the demand: The demand flow is piecewise constant. Between 5:58 and 6:02 the demand is 10, between 6:02 and 6:06 the demand is 20, and so forth. The interval between 6:38 and 6:42 is the first interval, where the demand exceeds the vehicle capacity, the interval between 7:14 and 7:18 is the last one, where the vehicle capacity is exceeded. The passengers, who have a desired departure time between 6:38 and 7:18 are expected to react to the insufficient capacity.

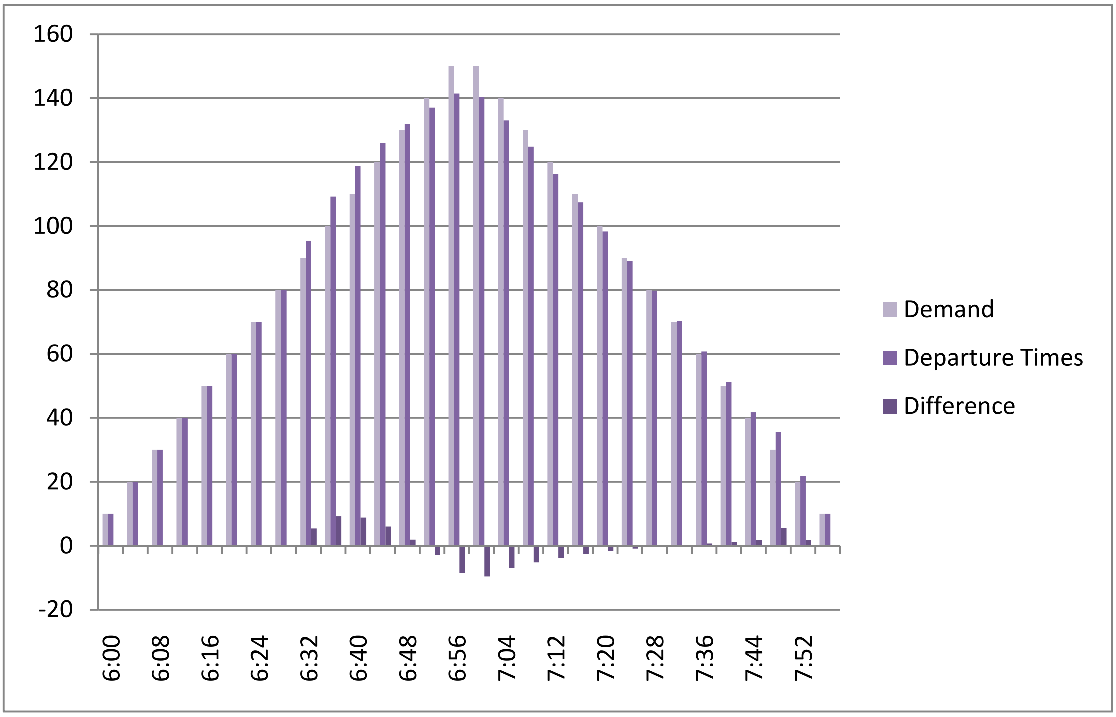

Figure 3: Assignment Result for Example 1

The result of the assignment procedure is also shown in Figure 3. In the peak time there are runs, where the demand exceeds the capacity. Passengers, who fail to board, have to wait for the next vehicle. Still, there are more passengers, who depart in the peak time than can be accommodated by the vehicles. Coming in the peak time and risk failing to board a vehicle is still more attractive for most passengers than starting the journey before or after the peak. The difference curve in Figure 3 shows that the actual demand for the runs between 6:32 and 6:48 is higher than the theoretical demand, because there are some passengers, who start their journeys earlier to avoid the congestion in the peak time. For the same reason, the runs between 6:52 and 7:28 have lower demand. The actual demand for the runs between 7:40 and 7:52 again is higher than the theoretical demand. The additional passengers using these runs try to avoid the queue that has formed in the peak time. The size of the queue can be seen in Figure 4. Its size increases until 7:16, then it decreases again.

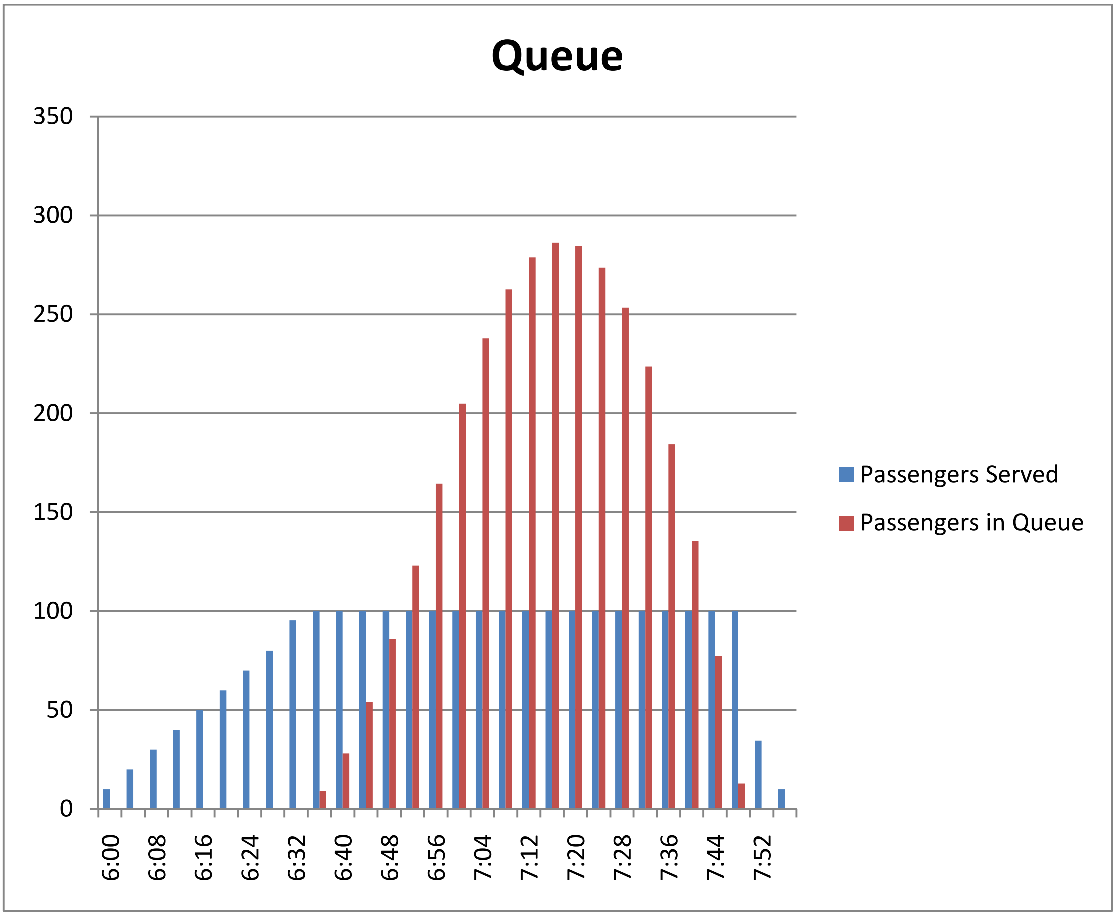

Figure 4: Queue in Example 1

The network is overcrowded in the peak time, although there would be sufficient capacity if passengers were more willing to change their departure times. The example is not supposed to show that passengers are unwilling to change their departure times. It rather shows that if changing the departure time for one second is as unattractive as waiting for a second in a crowded station, departure times do not change much.

5.2 Two Lines, Identical Headways, Same Departure Times

Example 2 is based on Example 1. In Example 2 there exists an additional Line 2 that always runs 1 minute after Line 1. Line 2 is slower; it takes 15 minutes to go from Stop 1 to Stop 2, but the vehicles have a large capacity (in the model it is assumed that the capacity is infinite). Passengers, who fail to board a fast vehicle of Line 1, have to choose between boarding the next slow vehicle and waiting for the next fast vehicle. Notice that at the moment when a slow vehicles arrives, a passenger, who decides to wait for the next fast vehicle has an expected remaining travel time of 13 minutes (3 minutes waiting plus 10 minutes travelling), if the next fast vehicle is reliable. Intuitively, the next fast vehicle needs to have quite a high reliability for the passenger to discard the slow vehicle. The information, how reliable the second fast vehicle is, is therefore very valuable to the passenger.
Table 1 shows the reliabilities of the vehicles that are not reliable. They all belong to the fast line.

Departure Time	Reliability
06:36	0.9
06:40	0.8
06:44	0.7
06:48	0.6
06:52	0.5
06:56	0.5
07:00	0.5
07:04	0.5
07:08	0.5
07:12	0.5
07:16	0.5
07:20	0.5
07:24	0.5
07:28	0.6
07:32	0.7
07:36	0.9

Table 1: Reliabilities of the Unreliable Vehicles

Table 2 shows the strategy costs for schedule-based and headway-based navigation.

Departure Time	TT sched.-based [s]	TT hdwy-based [s]	Difference [s]
06:41:00	670	900	230
06:44:00	708	708	0
06:45:00	708	900	192
06:48:00	744	744	0
06:49:00	744	900	156
06:52:00	780	780	0
06:53:00	840	900	60
06:56:00	780	780	0
06:57:00	840	900	60
07:00:00	780	780	0
07:01:00	840	900	60
07:04:00	780	780	0
07:05:00	840	900	60
07:08:00	780	780	0
07:09:00	840	900	60
07:12:00	780	780	0
07:13:00	840	900	60
07:16:00	780	780	0
07:17:00	840	900	60
07:20:00	780	780	0
07:21:00	840	900	60

07:24:00	780	780	0
07:25:00	840	900	60
07:28:00	728	728	0
07:29:00	728	859	131
07:32:00	679	679	0
07:33:00	679	804	125
07:36:00	624	624	0
07:37:00	624	780	156
07:40:00	600	600	0

Table 2: Strategy Costs for Example 2

At the departure times of the fast vehicles (the even times) schedule-based and headway-based navigation result in the same strategies. Up to 7:25, all passengers prefer boarding the slow line to waiting for the fast line. After 7:28 all passengers prefer waiting for the fast line. At the odd times schedule-based navigation is more successful, because passengers leave earlier than their desired departure time. They know that a fast vehicle is leaving just one minute earlier. Passengers navigating based on headways do not know this, and therefore their strategies have higher costs.

5.3 Two Lines, Identical Headways, Different Departure Times

Example 3 is similar to Example 2. The difference is that the slow line runs three minutes later than the fast line, not one minute. This makes the slow line less attractive for passengers, who navigate based on the schedule. These passengers know that when they skip the slow vehicle, they have to wait only 1 minute until the next fast vehicle arrives. Furthermore, the strategy to leave before the desired departure time is not as attractive as in Example 2, because passengers would have to leave 3 minutes early. Passengers navigating based on headways do not know, that the next fast vehicle after a slow vehicle runs just one minute later. They tend to overestimate the headway for the fast line. Therefore, in many cases they prefer the slow line to waiting for the next fast vehicle.
The reliabilities of the vehicles are the same as in Example 2.
Passengers navigating based on the schedule always prefer to wait for fast vehicles in this example. They never board slow vehicles. Passengers navigating based on headways, however, board the slow vehicles arriving between 6:43 and 7:27. Table 3 shows the travel times of each passenger type, and the difference between the travel times.

Departure Time	TT sched.-based [s]	TT hdwy-based [s]	Difference [s]
06:40:00	674	696	22
06:43:00	790	900	110
06:44:00	730	744	14
06:47:00	852	900	48
06:48:00	792	792	0
06:51:00	840	900	60

06:52:00	840	840	0
06:55:00	840	900	60
06:56:00	840	840	0
06:59:00	899	900	1
07:00:00	839	840	1
07:03:00	898	900	2
07:04:00	838	840	2
07:07:00	896	900	4
07:08:00	836	840	4
07:11:00	893	900	7
07:12:00	833	840	7
07:15:00	826	900	74
07:16:00	826	840	14
07:19:00	812	900	88
07:20:00	812	840	28
07:23:00	784	900	116
07:24:00	784	840	56
07:27:00	788	900	112
07:28:00	728	728	0
07:31:00	679	739	60
07:32:00	679	679	0
07:35:00	624	684	60
07:36:00	624	624	0
07:39:00	600	660	60
07:40:00	600	600	0

Table 3: Strategy Cost for Example 3

The largest difference in travel times is at 7:23, when passengers navigating based on the schedule save an expected 116 seconds compared to passengers navigating based on headways. One should notice, that the passengers navigating based on the schedule may have to wait for a very long time until they can board. A passenger arriving at 6:36 may have to wait until 7:40 until he can board a vehicle, although this is very unlikely. Nevertheless, it seems that this does not reflect normal behaviour. The model might be improved by either letting passengers board a vehicle after they have failed to board a vehicle at the same stop three times, for example. Another possibility is to let passengers choose the slow, reliable connection after three failures, even if they think that waiting for the next unreliable vehicle is more attractive.

6 CONCLUSIONS

The presented ideas are a first step to model passengers in schedule-based networks, who navigate based on headways. The difficulties of transferring the well-known static headway-based route choice models to the dynamic schedule-based network were highlighted: The cost, reliability, and frequency of lines change over time. In order to determine the attractive set of lines of a passenger at any given moment, cost, reliability and frequency of each line have to be defined for any given moment.

The model is only a first step. There exist various other ways to make headway-based choices, especially when there is real-time information at the stops (Gentile, Nguyen & Pallottino, 2005; Nökel & Wekeck, 2007). The given examples showed that having exact information about reliabilities and headways is a reasonable improvement to travelling based on estimated average headways. However, care should be taken when transferring this finding to the general case; the given examples seem to be too simple yet to come to a general conclusion.

One should also notice that interpreting the schedule-based strategies as strategies given by a portable device, while headway-based strategies are the ones passengers can "calculate" themselves, has one drawback: The presented strategy choice model is pre-trip. This means that the choices are made based on reliabilities as they are known before the trip starts. The real benefit of a portable device would be, that the reliabilities are frequently updated during the trip, and that the strategy is updated and optimized accordingly. Furthermore, departure times of vehicles could be different than in the schedule. Using this information would also lead to better strategies.

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