

ALTERNATIVE GRAVITY MODELLING APPROACHES FOR TRIP MATRIX SYNTHESIS

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ABSTRACT

An important stage in the development of highway assignment models is the development of a matrix of the number of trips going from each origin to each destination. A gravity model is one of the approaches for synthesising trip matrices, which uses a concept of the deterrence function, namely, it assumes that the interaction between two sites declines with increasing distances, cost and time of travel but is positively correlated with the amount of activity at each site. We report on a research programme on the process of calibrating a gravity model using different deterrence functions and constraints and propose to use a deterrence function based on a log-normal distribution as an alternative to the commonly used Tanner, exponential or power functions. We also discuss the application of sector level movements as constraints in a three-dimensional Furness which allows the synthetic matrices to be controlled to the partial trip matrices derived from roadside interview survey data. This approach is currently being implemented in the London Sub-regional highway assignment models.

This paper is concerned with use of a gravity model for the synthesis of base year trip matrices which are subsequently subject to various adjustments to match counts in order to achieve a satisfactorily validated highway assignment model. The models discussed in this paper are not intended to be used for forecasting future year demands or demand responses of highway interventions.

The opinions expressed here are those of the authors and not necessarily those of the Transport for London or Halcrow Group Limited.

Keywords: Gravity modelling, Trip distribution modelling, Deterrence function, Log-normal distribution, OD matrix

1. INTRODUCTION

Trip distribution models synthesise matrices of trips going from each origin to each destination. They can be generated from analogies with the physical world (e.g. gravity, electrostatic and opportunities), entropy-maximisation techniques, and

statistical techniques such as maximum likelihood estimation (MLE). The suitability of the model depends on the acceptability of the assumptions necessary for creation of the model and their interpretation as well as calibration and validation requirements.

The trip distribution problem can be expressed as a linear programming (LP) problem, which is a mathematical technique for determining a way to attain the best solution in a specified mathematical model for some list of requirements (called constraints) represented as linear equations (Evans, 1973). The form of the LP problem is as follows:

$$\min Z = \sum_{ij} T_{ij} C_{ij}, \text{ subject to } \sum_i T_{ij} = D_j, \sum_j T_{ij} = O_i,$$

where Z is the function to be minimised, T_{ij} are trips between origin i and destination j , C_{ij} is the cost of travel between origin i and destination j , O_i is the total number of trips originating at i , D_j is the total number of trips destined for j . The solution to this problem can be found using the method of Lagrange multipliers. Although the Linear Programming model may be a good approach for the behaviour of a single firm/client, it cannot represent aggregate behaviour for various trip makers.

The gravity model represents an alternative approach (Wilson, 1967¹) for solving the trip distribution model, which assumes that the interaction between two sites declines with increasing distance, cost and time of travel but is positively correlated with the amount of activity at each site. The rate of decline of the interaction, the “deterrence function”, has to be empirically determined. The model is doubly constrained so that total trips from zone i and total trips to zone j are equal to total origins and destinations respectively. The aim of the gravity model is to estimate or calibrate a deterrence function such that the synthesized trip cost distribution closely matches the trip cost distribution resulting from the road site interviews (RSIs), usually in terms of mean and standard deviation, and often, such that the synthesized sector-to-sector movements closely match the observed sector-to-sector movements.

Mathematically, the gravity model can be expressed as follows:

$$T_{ij} = A_i B_j O_i D_j f(C_{ij}), \sum_i T_{ij} = D_j, \sum_j T_{ij} = O_i, \text{ and}$$

$$A_i = \frac{1}{\sum_j B_j D_j f(C_{ij})}, B_j = \frac{1}{\sum_i A_i O_i f(C_{ij})},$$

where T_{ij} are trips between origin i and destination j ; C_{ij} is the cost of travel between origin i and destination j ; O_i is the total number of trips originating at i ; D_j is the total number of trips destined for j ; A_i , B_j are balancing factors solved iteratively; and $f(C_{ij})$ is the distance decay factor.

Common forms for the distance decay (or deterrence) functions are

¹ First use of entropy maximisation technique, leading to balancing factors as currently known

- exponential function: $f(C_{ij}) = \exp(-\beta C_{ij})$,
- power function: $f(C_{ij}) = C_{ij}^\alpha$, and
- a combined function (Tanner function): $f(C_{ij}) = C_{ij}^\alpha \exp(-\beta C_{ij})$,

where α and β are parameters to be calibrated.

In the limit of $f(C_{ij}) \rightarrow 1$ (Furness, 1965) the gravity model will produce a matrix of movements independent of transport costs (the Furness solution), whereas $f(C_{ij}) \rightarrow 0$ will generate a solution closer to the LP solution, i.e. where transport costs are dominant. Despite the fact that the gravity model is very successful in explaining the choice of a large number of trip makers, the choice of any individual trip maker can show a great discrepancy from the predicted value.

The gravity model synthesises trip patterns based on only inter-zonal distances or generalised costs at an aggregate level and it cannot take into account other factors which may affect individual decisions; the importance of such factors is reduced on aggregation. However, there may be pairs of zones which have a particular relationship in terms of trip making, and as a result, more or less trips will take place between these two locations than predicted by the gravity model. A number of options are available to solve this problem including:

- introduction of an additional set of parameters K_{IJ} to the gravity model which apply to sector-to-sector movements and which scale trips T_{ij} , $i \in I$, $j \in J$, by the value of the K-factor K_{IJ} ;
- introduction of an additional set of parameters L_{IJ} to the gravity model which apply to sector-to-sector movements and which scale costs C_{ij} , $i \in I$, $j \in J$, by the value of the L-factor L_{IJ} ;
- introduction of an additional set of parameters F_{IJ} at a sector-to-sector level to the final matrices to ensure that the synthesised matrices match the observed matrices at a sector-to-sector level;
- calibration of separate deterrence functions for different movements;
- creation of additional purposes associated with the specific movements and calibration of separate deterrence functions for those purposes; or
- a combination of the above.

We report on a research programme on the process of calibrating a gravity model using different deterrence functions and constraints. In section 2, we specify alternative procedures for incorporating additional influences into the model, describe the main tests which have been undertaken during the course of model calibration, and set out and contrast the key results obtained. Section 3 describes the main results whilst Section 4 concludes.

This paper is concerned with use of a gravity model for the synthesis of base year trip matrices which are subsequently subject to various adjustments to match counts in order to achieve a satisfactorily validated highway assignment model. The models

discussed in this paper are not intended to be used for forecasting future year demands or demand responses of highway interventions.

2. GRAVITY MODELLING - TESTING PROGRAMME

We have undertaken an intensive research programme on the process of calibrating a gravity model using TfL's highway assignment model NoLHAM of the London north sub-region and our gravity modelling software, an Excel spreadsheet, coded in Microsoft Visual Basic for Applications. Alternative procedures for incorporating additional influences into the model (the fact that some movements are more or less attractive, independent of the travel cost) were considered. The following series of tests have been carried out.

- The classical Tanner function with or without sectoral constraints - these tests used a Tanner function and the aim of the tests was to find alpha and beta parameters which result in the best matching of the trip cost distributions for each purpose: $f(C_{ij}) = C_{ij}^{\alpha} \exp(-\beta C_{ij})$.

Note that the form of the Tanner function is similar to the density function of the gamma distribution².

- An exponential function with sectoral constraints – in these tests an exponential function ($\alpha = 0$) was used to find the best fit to the trip cost distributions. The functional form was:

$$f(C_{ij}) = \exp(-\beta C_{ij})$$

and only one coefficient, β , was to be calibrated. The objective function should have an equal or higher value (no better fit) than with tests based on the Tanner function.

- A power function with sectoral constraints – in these tests a power function ($\beta = 0$) was used to find the best fit to the trip cost distribution for each purpose. The functional form in this case was:

$$f(C_{ij}) = C_{ij}^{\alpha}$$

and only one coefficient α , α , was to be calibrated. As with the previous set of tests, the objective function should have an equal or higher value (no better fit) than with tests based on the Tanner function.

- The probability density function of a log-normal distribution with or without sectoral constraints – a gravity function in the form of the probability density function of a log-normal distribution was implemented. In this case:

²The equation stating the probability density function of a gamma-distributed random variable x is

$$f(x, \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta x) \text{ for } x \geq 0,$$

where α and β are the shape and inverse scaling parameters respectively. If α is a positive integer, $\Gamma(\alpha)$ can be calculated as $\Gamma(\alpha) = (\alpha - 1)!$ Otherwise, $\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} \exp(-t) dt$.

$$f(C_{ij}) = \frac{1}{C_{ij} \cdot \sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\ln C_{ij} - \mu)^2}{2\sigma^2}}$$

where σ and μ are the coefficient to be calibrated (note that μ is the mean and σ is the standard deviation of the log-normal distribution we will fit).

Without the sectoral constraints, the model was constrained only to production and attraction totals. In tests with sectoral constraints, the synthetic matrices were constrained to productions, attractions and reliable sector-to-sector movements totals using a three-dimensional Furness procedure³, and Tanner or log-normal functions were used as a gravity function in the initial set up of the matrix of trips (matrix “seeding process”) for those tests. The order of the balancing procedure was origins, destinations and sectors. Since sectors were balanced last, the distribution matched observed sector to sector movements exactly. The balancing was applied only to the sector to sector movements which were regarded as being statistically reliable. A threshold of a 95% confidence interval within +/- 30% of the cell value was used first. At this level, a reasonable number of cell values were regarded as sufficiently reliable to be used as constraints on the synthesised trip matrices. In further tests the threshold was changed to +/- 10% and +/-50%.

We also tested an option of splitting a trip cost distribution into two parts which represent ‘low’ and ‘high’ costs, and calibrating two Tanner functions in order to find the best fit to the trip cost distributions. Two synthetic matrices were produced for each purpose as a result, one for ‘high’ and one for ‘low’ trip costs:

$$\begin{aligned} f(C_{ij}) &= C_{ij}^{\alpha_L} \exp(-\beta_L C_{ij}) & \text{for } C_{ij} \leq \text{Cut-Off GC} \\ f(C_{ij}) &= C_{ij}^{\alpha_H} \exp(-\beta_H C_{ij}) & \text{for } C_{ij} > \text{Cut-Off GC} \end{aligned}$$

A cut-off point for the generalised cost was tested. We found that the level of discontinuity produced around the trip costs cut point was usually quite substantial, and therefore additional smoothing was normally required. A procedure, based on using weighting parameters which act to smooth the ‘join’ between the two trip cost distributions, can be implemented if such an approach is chosen⁴.

³The Furness procedure is also known as iterative proportional fitting procedure (IPF) or matrix scaling procedure in mathematics, statistics and computer science. This procedure generates a sequence of improving approximate solutions (subject to convergence) for a number of problems by scaling data cells (non-zero) so that they add up to selected totals for both the columns and rows (in the two-dimensional case) and any additional totals (often called marginal totals). Note that the marginal column totals and the marginal row totals must sum to the same value for IPF to work.

⁴ The following procedure was implemented in the first version of West London highway model called WelHAM but it was not implemented in the testing programme due to the aim of the tests to look at the trip cost distribution fit. The aim of the procedure is to replicate the total number of observed trips whilst minimising the difference between the partial matrix and synthetic matrix for fully observed movements. The weights x_1 and x_2 can be found that satisfy the following equation⁴:

$$T = x_1 \cdot T_{LowAll}^{Syn} + x_2 \cdot T_{HighAll}^{Syn},$$

The fit to a trip cost distribution can be improved further if additional weights are implemented to smooth effects of generalised costs. In the first versions of the London sub-regional highway assignment models, a set of L-factors, as discussed in the previous section, was applied. This was done in a discrete manner and values of L-factors were significantly different from 1, which raised concerns of the boundary effects around areas with different L-factors.

As part of the testing programme, we investigated options of continuous smoothing the generalised costs by applying a distance-deterrence function, as there is some evidence that the sensitivity of demand responses to change in generalised cost reduces with increasing trip length (Dale, 2008). To model this, we tested cost damping functions in the form of a negative logistic function as specified in the UK Department for Transport TAG Unit 3.10.2, Section 1.11.

A negative logistic function used is represented by the equation:

$$d_{ij} = \frac{\gamma + e^{(\zeta \cdot C_{ij} + k)}}{1 + e^{(\zeta \cdot C_{ij} + k)'}}$$

where d_{ij} is the deterrence function of the distance from origin i to destination j , and γ, ζ and k are calibration coefficients. Parameter ζ ($\zeta < 0$) defines how sharply the curve moves from the upper limit (always 1 as $\gamma \leq 1$) to the lower limit ($\gamma \geq 0$) value of the range. Parameter k defines the “lag” distance, or “cut-off” distance, namely, when the curve starts to move from the upper limit to the lower limit.

The functional form in this case was:

$$f(C_{ij}) = (d_{ij} \cdot C_{ij})^\alpha \cdot e^{\beta \cdot (d_{ij} \cdot C_{ij})},$$

and the coefficients to be calibrated are $\alpha, \beta, \gamma, \zeta$ and k .

If the probability density function of a log-normal distribution was used, the functional form of the gravity function is then:

$$f(C_{ij}) = \frac{1}{d_{ij} \cdot C_{ij} \cdot \sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\ln(d_{ij} \cdot C_{ij}) - \mu)^2}{2\sigma^2}}$$

and the coefficients to be calibrated were $\mu, \sigma, \gamma, \zeta$ and k .

where T is the total target number of trips, T_{LowAll}^{Syn} is the total of low cost synthetic matrix below the chosen threshold, $T_{HighAll}^{Syn}$ is the total of high cost synthetic matrix above the chosen threshold; and minimise the function

$$f(x_1, x_2) = (T_{Low}^{observed} - x_1 \cdot T_{LowObserved}^{Syn})^2 + (T_{High}^{observed} - x_1 \cdot T_{HighObserved}^{Syn})^2,$$

where $T_{Low}^{observed}$ is partial matrix total below the threshold, $T_{High}^{observed}$ is partial matrix total above the threshold, $T_{LowObserved}^{Syn}$ is the total number of synthetic matrix total for observed movements below the threshold, and $T_{HighObserved}^{Syn}$ is number of synthetic matrix total for observed movements above the threshold.

In the case of the WebTAG distance-based cost-damping function used, the gravity functions were in the following forms for a Tanner and log-normal distribution-based functions respectively:

$$f(C_{ij}) = ((l/\eta)^{-\vartheta} \cdot C_{ij})^{\alpha} \cdot e^{\beta \cdot (l/\eta)^{-\vartheta} \cdot C_{ij}}.$$

where l was the average trip length, and α, β, η and ϑ were the coefficients that were calibrated, and

$$f(C_{ij}) = \frac{1}{(l/\eta)^{-\vartheta} \cdot C_{ij} \cdot \sigma \cdot \sqrt{2\pi}} \cdot e^{-\frac{(\ln(l/\eta)^{-\vartheta} \cdot C_{ij}) - \mu)^2}{2\sigma^2}}$$

where μ, σ, η and ϑ were the coefficients that were calibrated.

In order to run the gravity model, the following inputs were required: an observed trip matrix (that is, a trip matrix derived from roadside interview surveys, often known as a “partial” trip matrix), trip ends (either origins and destinations or productions and attractions), a matrix which defines the “observed” and “unobserved” movements, a matrix of sectoral constraints, a cost matrix, an upper bound limit for the distribution analysis and a cost interval – a trip cost distribution was aggregated to the range given by the ratio between the limit and the interval.

The implemented gravity modelling approach can be summarised as follows.

- Step 1: Choose a range of possible parameters (depending on a function used, the parameters are $\alpha, \beta, \gamma, \zeta, k, \eta, \vartheta, \mu$, and σ). Choose a step for evaluating possible parameters values. Run an exhaustive search algorithm by checking all values of the parameters from the chosen interval with a chosen step and finding a set of parameters which minimises the value of the sum of squares of the absolute differences between the synthesised and observed distributions: $e_{parameters} = \sum_c (T_c^{obs} - T_c^{synth})^2$, where T_c^{obs} is the number of observed trips in trip cost band c ; and T_c^{synth} is the number of synthesised trips in trip cost band c .
- Step 2: Make $K_{IJ} = 1$ for each I, J ; $A_i = 1$ for each i , $B_j = 1$ for each j and balance the typical doubly constrained gravity model as follows $T_{ij} = A_i B_j O_i D_j f(C_{ij})$, $\sum_i T_{ij} = D_j$, $\sum_j T_{ij} = O_i$, and

$$A_i = \frac{1}{\sum_j B_j D_j f(C_{ij})}, B_j = \frac{1}{\sum_i A_i O_i f(C_{ij})}.$$

If $\max \left\{ \max_i \left(\frac{O_i^{l+1} - O_i^l}{O_i^{l+1}} \right), \max_j \left(\frac{D_j^{l+1} - D_j^l}{D_j^{l+1}} \right) \right\} \leq \varepsilon$, where $l+1$ and l denote the

current and previous iteration numbers respectively, or if $l+1 = l_{max}$, the maximum number of iterations, then stop. Otherwise increase l by one and balance rows and columns again.

- Step 3: Compress the trips resulted from applying the gravity model T_{ij} to the study area sector system

$$T_{KIJ}^{Syn} = \sum_{KIJ} T_{ij}.$$

Compress the observed trips, $T_{ij}^{Observed}$, to the sector system

$$T_{KIJ}^{Observed} = \sum_{KIJ} T_{ij}^{Observed}$$

Calculate the K factors:

$$K_{IJ} = \frac{T_{KIJ}^{Observed}}{T_{KIJ}^{Syn}} \text{ if there is confidence in } IJ, \text{ otherwise } K_{IJ} = 1.$$

Balance the gravity model with K-factors:

$$T_{ij} = A_i B_j K_{IJ} O_i D_j f(C_{ij}), \sum_i T_{ij} = D_j, \sum_j T_{ij} = O_i, \text{ and}$$

$$A_i = \frac{1}{\sum_j B_j D_j K_{IJ} f(C_{ij})}, B_j = \frac{1}{\sum_i A_i O_i K_{IJ} f(C_{ij})}.$$

Table 1. Calibration Parameter Search Summary

Test	Parameter	Lower Bound	Upper Bound	Search Step	Loops	Total Loops
Tanner	α	-0.75	0.25	0.05	21	441
	β	-0.5	0	0.025	21	
Log Normal	σ	0.5	5	0.5	10	100
	μ	0.5	5	0.5	10	
WebTag Cost Damping	α	-1	1	0.25	9	585
	β	-0.24	0	0.02	13	
	β_{CD}	-0.9	-0.1	0.2	5	
Low / High Cost	α_L	-0.75	0	0.25	4	2704
	β_L	-0.24	0	0.02	13	
	α_H	-0.75	0	0.25	4	
	β_H	-0.24	0	0.02	13	

The model was calibrated using an exhaustive search algorithm to find the parameters. The calibration criteria used was the minimum sum of square differences⁵ between the observed and the synthetic trip cost distributions, practitioners usually calibrate gravity models in terms of means and standard

⁵ It can be argued that an MLE would be better; however, minimising the square difference is almost equivalent and has computational advantages especially taking into account the size of the London Sub-Regional models (around 2000-4000 zones depending on a region).

deviation, these figures were also checked and in general minimum sum of squares would produce satisfactory synthetic means and standard deviations. The search for optimum parameters was normally done within the ranges and steps shown in Table 1. Checks for local optima were also required. If further refinement of a parameter was needed, it was normally done around the previous optimum and using a 10th of the search step.

3. RESULTS

This section presents results of the undertaken research programme. We ran 25 major tests based on four gravity functions, namely Tanner, exponential, power and log-normal. We investigated the impact of the sectoral constraints with a threshold of a 95% confidence interval within +/-30%, +/-10% and +/- 50% of the cell value. We also tested the impact of continuous smoothing of the generalised costs by applying a distance-deterrence function in the form of either the negative exponential function or a negative logistic function as described in Section 2. Each test was run for 6 purposes available in NoLHAM (HBW – Home-Based Work, HBEB – Home-Based Employer Business, HBEd – Home-Based Education, HBO – Home-Based Other, NHEB – Non Home-based Employer Business, NHBO – Non Home-Based Other) or for two combined purposes (Home-Based Business and Home-Based Other).

The alternative model forms were explored using the full set of six purposes. The key performance indicators used were:

- the sum of the squares of the differences between observed and synthesised numbers of trips in each cost bin in the trip cost distributions;
- the comparisons of the observed and synthesised mean trip costs and lengths; and
- and the comparisons of the standard deviations of the observed and synthesised trip cost and length distributions.

The best fit for all purposes was achieved using the probability function of a log-normal distribution with K-factors. The Tanner function with K-factors, separate Tanner functions with K-factors for low and high costs, and the Tanner function with K-factors and WebTAG distance-based cost-damping function performed quite well. The exponential function with K-factors and the Tanner function with a negative logistic distance deterrence function performed less well in general, and the power function with K-factors performed poorly. As an example, Figure 1 shows the results of a number of tests for the Home-Based Other (HBO) purpose. The fit for other purposes was similar with the exception of the Home-Based Education (HBEd) purpose which, due to a spiky observed trip cost distribution, was more difficult to fit (Figure 2 shows the best match achieved using a log-normal function.) As a result of the application of the K-factors in a 3D (three-dimensional) Furness process, the parameter values and fits to the observed trip cost and length distributions changed but not significantly. The overall fit to the observed distribution was usually a bit worse due to additional constraints implemented, whilst the key performance indicators changed differently for different purposes and not necessarily all indicators for the same purpose changed in the same direction.

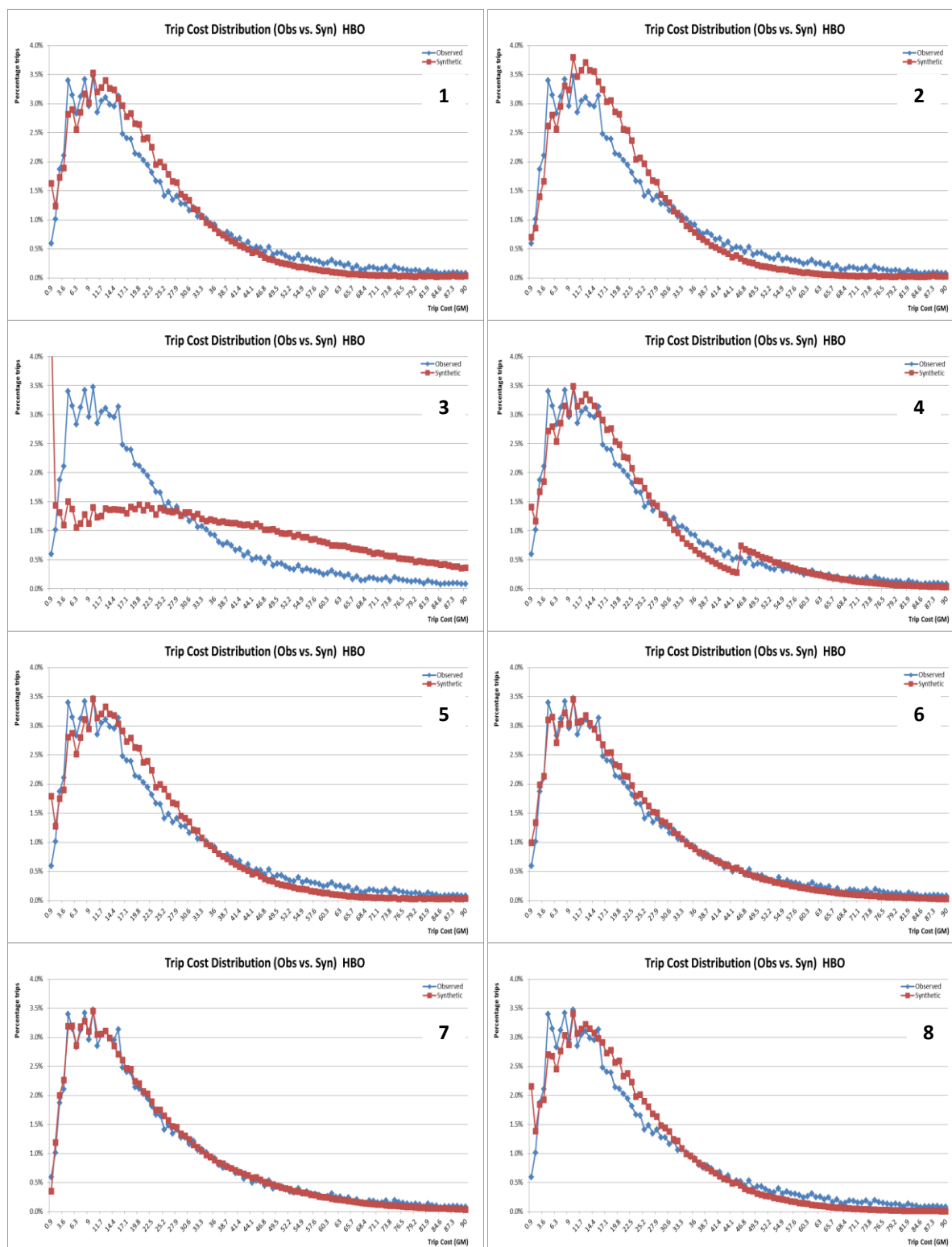


Figure 1. Observed and synthetic trip cost distributions, purpose: home-based other (HBO). Percentage number of trips versus generalised costs (minutes). Results of the gravity modelling using 1) a Tanner function and K-factors; 2) an exponential function and K-factors; 3) a power function and K-factors; 4) two Tanner functions for long and short distances, no optimisation, and K-factors; 5) a Tanner function with a distance deterrence function and K-factors; 6) a Tanner function with WebTAG cost-damping function (a function of distance) and K-factors; 7) a log-normal function and K-factors; 8) a Tanner function and 3D-Furness.

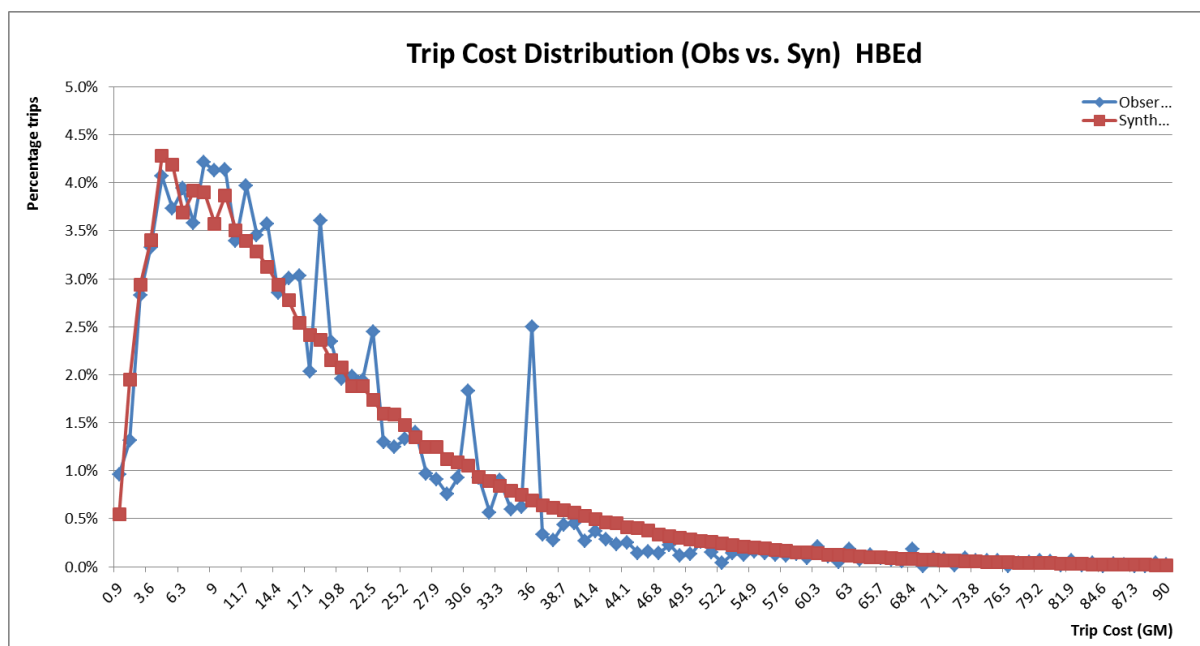


Figure 2. Observed and synthetic trip cost distributions, purpose: home-based education (HBEd). Percentage number of trips versus generalised costs (minutes). Results of the gravity modelling using a log-normal function and K-factors.

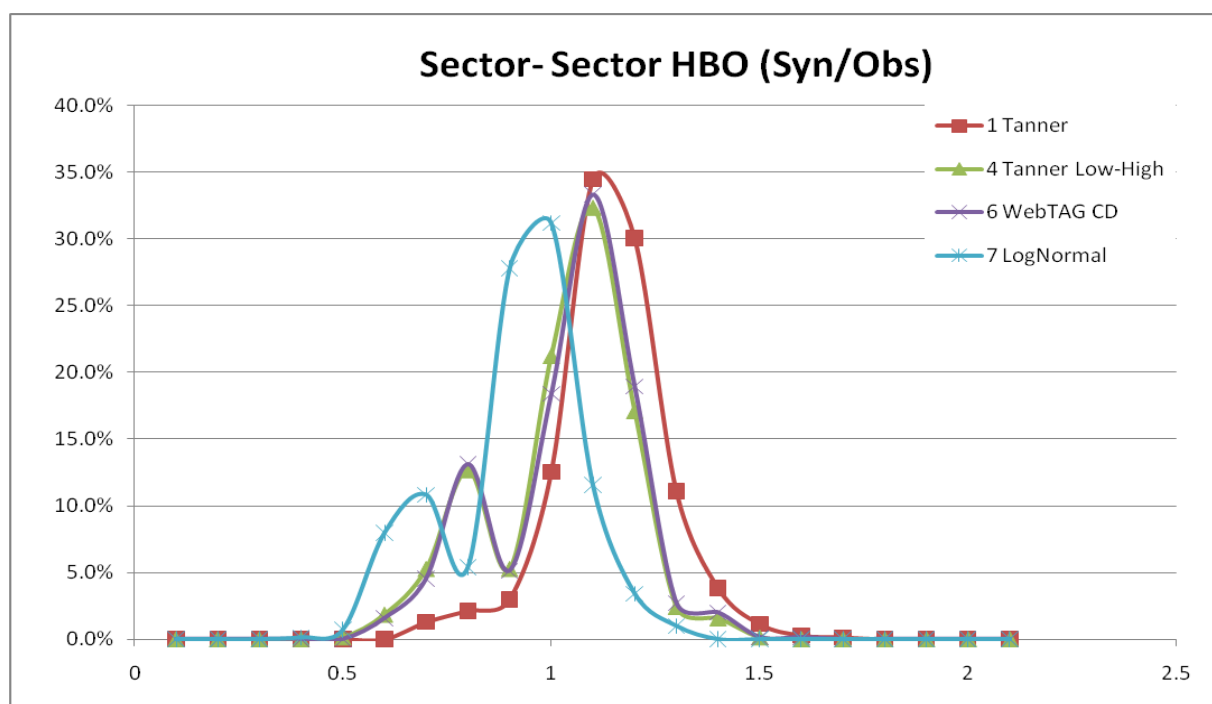


Figure 3. Frequency distributions of the differences between the observed and synthesised movements, purpose: home-based other (HBO). Results of the gravity modelling using 1) a Tanner function and K-factors; 4) two Tanner functions for long and short distances, no optimisation, and K-factors; 6) a Tanner function with WebTAG cost-damping function (a function of distance) and K-factors; 7) a log-normal function and K-factors.

Figure 3 illustrates frequency distributions of the differences between the observed and synthesised movements for four the better-performing models. These distributions show that all four models run without the use of K-factors did not reproduce the observed movements sufficiently well.

Table 2 and Table 3 show the impact of the K-factors on the log-normal gravity function parameter values, and also on the mean, standard deviation and the sum of square of differences between the synthetic and observed number of trips in each cost bin. The effect of the K-factors on the parameter values and the key indicators is not large, and the sectoral constraints improve the key indicators for some purposes but worsen for others.

Table 2. Key performance indicators, log-normal function *with* K-factors

<i>Purpose</i>	<i>Gravity Function Parameters</i>		<i>Sum of Square Differences</i>	<i>Cost (min)</i>			
	σ	μ		Observed Mean	Synthetic Mean	Observed Deviation	Synthetic Deviation
HBEB	1.175	1.773	1.962	30.45	29.94	19.66	18.51
HBEd	0.953	1.248	10.011	17.41	18.18	13.61	14.42
HBO	1.027	1.473	0.752	22.44	21.71	17.61	16.35
HBW	1.063	2.447	0.822	34.04	34.43	19.61	19.73
NHBEB	1.267	1.248	1.810	26.06	25.31	18.60	17.45
NHBO	0.978	1.527	2.092	22.34	22.12	17.17	16.28

Table 3. Key performance indicators, log-normal function *without* K-factors

<i>Purpose</i>	<i>Gravity Function Parameters</i>		<i>Sum of Square Differences</i>	<i>Cost (min)</i>			
	σ	μ		Observed Mean	Synthetic Mean	Observed Deviation	Synthetic Deviation
HBEB	1.075	1.775	1.93	30.45	29.52	19.66	18.19
HBEd	0.925	1.225	10.76	17.41	17.59	13.61	14.03
HBO	0.975	1.425	0.95	22.44	22.11	17.61	16.50
HBW	0.975	2.225	1.11	34.04	34.05	19.61	19.84
NHBEB	1.175	1.275	1.83	26.06	25.13	18.60	17.08
NHBO	0.925	1.525	2.19	22.34	21.71	17.17	15.72

Table 4 and Table 5 show the impact of the sectoral constraints with a threshold of a 95% confidence intervals within 10%, 30% and 50% of the cell value for two purposes, Home-Based Work and Home-Based Other, as an example. The Tanner function with K-factors was used in these tests. For each of the six purposes the parameter values changed from the values based on a 95% confidence interval of

30% of the cell value, noticeably more so in the case of the 10% threshold than with the 50% threshold. While the quality of the match between the observed and synthesised trip cost distributions did not change by an order of magnitude, the use of the 10% threshold did consistently produce a worse match than use of the 30% and 50% thresholds, and use of the 50% threshold generally produced the best fit to the observed trip cost distributions. The number of sectors used as sectoral constraint with the 50% threshold increased by about 2.5 times compared to the number of sectors used with the 10% threshold. However, with only this information available we cannot say which threshold gives the parameter values that synthesise the non-surveyed movements most accurately.

Table 4. Impact of the sectoral constraints with a threshold of a 95% confidence interval within +/-30%, +/-10% and +/- 50% of the cell value. Tanner function, with K-factors, purpose: Home-Based Work (HBW).

<i>Test</i>	<i>Deterrence Function Parameters</i>		<i>Sum of Square Diff.</i>	<i>Cost (min)</i>				<i>No of sectors used/maximum number of sectors</i>
	<i>A</i>	<i>β</i>		<i>Obs. Mean</i>	<i>Syn. Mean</i>	<i>Obs. Dev.</i>	<i>Syn. Dev.</i>	
+/- 10% CI	-0.100	-0.075	1.312	34.04	32.72	19.61	18.11	400 / 2025
+/- 30% CI	-0.200	-0.065	0.796	34.04	33.70	19.61	18.92	759 / 2025
+/- 50% CI	-0.250	-0.062	0.673	34.04	33.62	19.61	19.15	1008 / 2025

Table 5. Impact of the sectoral constraints with a threshold of a 95% confidence interval within +/-30%, +/-10% and +/- 50% of the cell value. Tanner function, with K-factors, purpose: Home-Based Other (HBO).

<i>Test</i>	<i>Deterrence Function Parameters</i>		<i>Sum of Square Diff.</i>	<i>Cost (min)</i>				<i>No of sectors used/maximum number of sectors</i>
	<i>A</i>	<i>β</i>		<i>Obs. Mean</i>	<i>Syn. Mean</i>	<i>Obs. Dev.</i>	<i>Syn. Dev.</i>	
+/- 10% CI	-0.400	-0.125	6.840	22.44	19.07	17.61	13.17	421 / 2025
+/- 30% CI	-0.350	-0.125	5.508	22.44	19.67	17.61	13.68	734 / 2025
+/- 50% CI	-0.325	-0.128	4.411	22.44	19.86	17.61	14.35	1053 / 2025

Implementation of the 3D Furness process results in the synthetic matrices being controlled to the sectoral constraints. The observed and synthesised movements at a sectoral level were compared for the four better-performing functions for each of the six purposes. Figure 4 presents the results for the Tanner function tests for the Home-Based Other purpose. The R^2 values of the regression lines in the cases where classic K-factors have been used all have values in excess of 0.938, although the slopes vary noticeably from one in some cases. This imperfect correlation between observed and synthesised values is a reflection of the imperfect nature of the K-factors. In contrast, the tests in which the sector level observed movements have been applied as a constraint in a 3D Furness all show R^2 values and slopes of one (as expected). Regression analyses of these comparisons for in-work and out-

of-work purposes show R^2 values in excess of 0.96 in all cases, and slopes reasonably close to one (the most divergent from one being 1.14), indicating that there is little to choose between the six-purpose and two-purpose approaches.

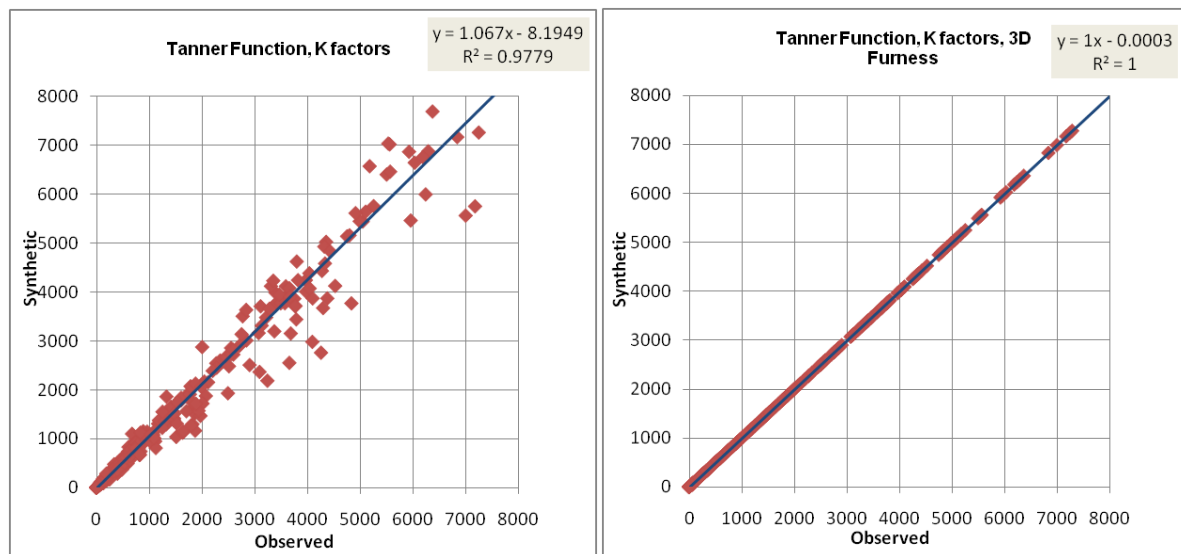


Figure 4. Regression analysis of sector-to-sector movements, observed versus synthetic, a Tanner function with K-factors, purpose: home-based other (HBO).

We compared the sectoral level movements for which values were not available from the survey data (“non-observed”) produced by models calibrated for six and two purposes based on the Tanner function, the Tanner function with TAG distance-based cost damping and the probability density function of the log-normal distribution, each with K-factors as well as with and without the application of the 3D Furness. Regression analyses of these comparisons for in-work and out-of-work purposes show R^2 values in excess of 0.96 in all cases, and slopes reasonably close to one (the most divergent from one being 1.14). An example of the results is shown in Figure 5.

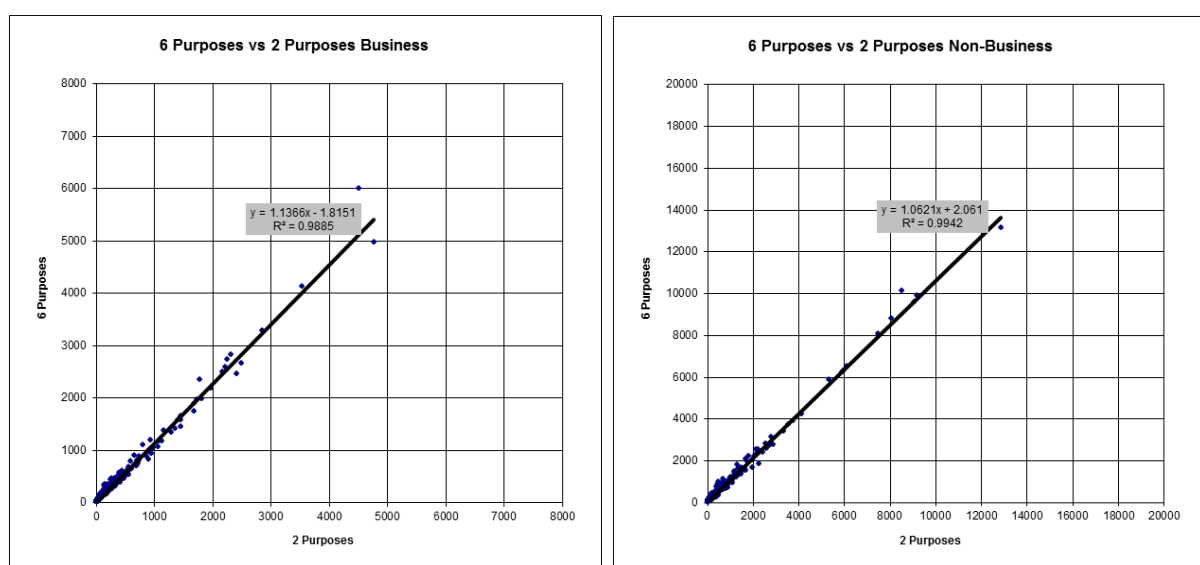


Figure 5. Comparison of results from 6-purposes run versus 2 purposes runs, a log-normal function, 3D-Furness, K-factors, purpose: in-work (left) and out-work (right).

Finally, we investigated the impact of extending the tail of the trip-cost distribution and applying weights to improve further a fit at the longer end of the distribution. (Note that the accuracy of the long-distance trip estimates can have significant impacts on the fuel cost elasticities in the model realism tests.) The results demonstrate that simply extending the tail of the trip cost distribution had a small effect in terms of the “sum of the square differences” measure, more so for in-work than out-of-work. However, applying weights at the longer end caused considerable deterioration in the fit to the distribution as a whole, as measured by the “sum of the square differences”, although the means and standard deviations were largely unaffected.

The computation time of different tests varied in accordance to the number of parameters in the function and the computation steps used. The Tanner and log-normal functions without any additional cost-damping functions required about the same computation time per a set of parameters tested; however, fitting a probability function of the log-normal distribution required less refinement of the steps as parameters seemed to be sensitive up to 0.5. The Tanner function parameters were more sensitive to the additional decimal places and a step of 0.001 was used for refining the parameters. The WebTAG cost-damping function increased the run time by at least five times due to an additional parameter used, and the use of two Tanner functions for low and high costs increased the run time by at least ten times.

4. CONCLUSIONS

The paper is concerned with use of a gravity model for the synthesis of base year trip matrices which are subsequently subject to various adjustments to match counts in order to achieve a satisfactorily validated highway assignment model. The models discussed in this paper are not intended to be used for forecasting future year demands or demand responses of highway interventions.

We have found that the Tanner function with K-factors, separate Tanner functions with K-factors for low and high costs, the Tanner function with K-factors and TAG distance-based cost damping function, and a probability function of a log-normal distribution with K-factors reproduced the observed trip cost distributions reasonably well. Without the use of K-factors, only some movements were accurately reproduced, although the overall fit to the trip cost distributions was good.

The use of a 3D Furness process to constrain the synthetic matrices to statistically reliable observed movements resulted in the synthesised and observed movements agreeing very closely, and without changing the model parameters significantly.

Extending the tail of the trip-cost distribution in order to improve further a fit at the longer end of the distribution had a small effect in terms of the “sum of the square differences” measure, more so for in-work than out-of-work. However, applying weights at the longer end caused considerable deterioration in the fit to the distribution as a whole. The means and standard deviations were largely unaffected.

The use of separate Tanner Functions for low and high costs offered no better performance than the other functional forms and had the disadvantage of requiring two calibrations instead of one, an optimisation procedure of finding the optimum point for splitting costs into low and high ranges, and the process for smoothing the

discontinuity between the two functions. Of the other three, the log-normal distribution-based function was the most efficient to compute and the Tanner Function with distance-based cost damping was the least efficient.

Applying the models for just two purposes, in-work and out-of-work, produced estimates of the non-observed sector level movements which were quite close to the estimates produced by working at the more usual six purpose level of segmentation.

We propose to use a deterrence function based on a log-normal distribution with observed movements applied in a 3D Furness process, which allows controlling the synthetic matrices to the partial trip matrices at the sectoral level. This approach is currently being implemented in the London Sub-regional highway assignment models.

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