Accessibility to Vacant Activities: a Novel Model of Destination Choice

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INTRODUCTION

The spatial distribution of trips from origin zones to destination zones is usually modelled by physical analogy (eg. gravity model) or by discrete choice from among destination zones. In order to improve the microeconomic interpretation, Koenig (1974) and Cochrane (1975) have introduced a model of choice from among individual activities, in which it is assumed that (i) all activities have independent, identically distributed random utilities, and (ii) an activity serviced to a consumer is still available to other ones.

The purpose of the paper is to put forward a novel model of choice from among individual activities, called AVA for Accessibility to Vacant Activities. In AVA it is assumed that each activity has a gross value same for all consumers, and that each consumer chooses the best vacant activity, i.e. the one with maximal net value after subtraction of the transport cost. Thus the distribution of trips from origins to destinations results from the individual assignment of activities to consumers.

The paper contains four parts. First AVA's assumptions are introduced and discussed. Then their consequences are analyzed in the binary case. The gravity model is obtained as a special case, assuming exponentially-distributed utilities for activities in every destination zone. Next we address the general case with several origin and destination zones; we formulate the model as the solution of a concave maximization program, of which the objective function represents the consumer surplus. This is further extended to elastic demand and congested transport. Lastly AVA is compared to other trip distribution models, with emphasis on the Koenig-Cochrane model.

1 ECONOMIC ASSUMPTIONS

Activity supply in a destination zone. Let us consider a destination zone, say *d*, in which A_d activities of a given purpose are supplied. Each activity *k* has a gross value of v_k to any potential consumer (eg. wages for work purpose, or the opposite of price for leisure purpose). We denote by H_d the CDF of the gross activity values in zone *d*. Thus the number of activities with gross value greater than *v* is $A_d(1-H_d(v))$. We also assume that every activity may be serviced to one consumer only.

Activity demand. Activity demand is the set of the potential activity consumers. A consumer located in an origin zone *o* derives from an activity *k* in zone *d* with gross value v_k a net

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value $u_k(o) = v_k - G_{od}$, in which G_{od} is the transport cost from o to d. We assume that each activity consumer is a rational economic decision-maker and strives to maximize the net value of his consumption.

Demand-supply equilibrium. By omitting the transport cost, we may focus on the local activity market of a single destination zone d. Assuming there are X_o consumers, this corresponds to the number of serviced activities, T_d, if $X_o \leq A_d$. Only those activities with high value may be consumed since every consumer maximizes their gross value (identical to net value in this case). Thus the equilibrium point of the market corresponds to quantity T_d and value $\tilde{v}_d$ such that $T_d = A_d(1 - H_d(\tilde{v}_d))$.

We further make the regularity assumption that the volumes check $0 \leq X_o \leq A_d$ and that the inverse function H_d^{-1} is continuous on $]0, 1[$. We also define $H_d^{-1}(0) = \inf\{v\,;\, H_d(v) > 0\}$ and $H_d^{-1}(1) = \sup\{v\,;\, H_d(v) < 1\}$.

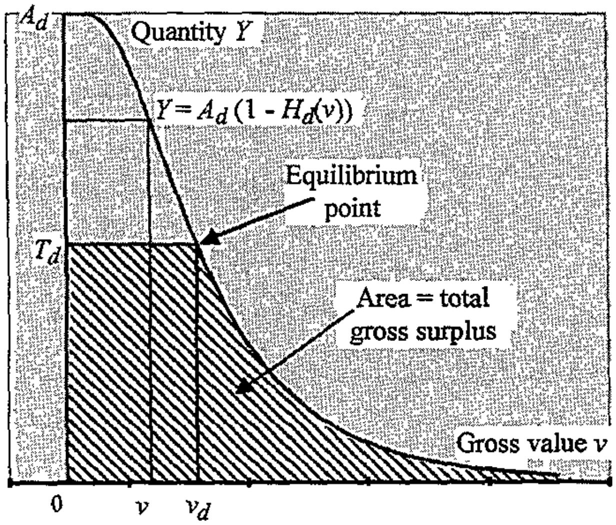
Then if $0 < T_d = X_o < A_d$ it holds that $\tilde{v}_d = H_d^{-1}(1 - T_d / A_d)$ and corresponds to both the minimum value of serviced activities and the maximum value of vacant activities: we call it the accessibility to vacant activities (AVA). If $T_d = 0$, then $\tilde{v}_d = H_d^{-1}(0)$ corresponds to only the maximum value of vacant activities, whereas if $T_d = A_d$, then $\tilde{v}_d = H_d^{-1}(1)$ corresponds to only the minimum value of serviced activities.

We can measure the total gross surplus of consumers by summing the gross values of the serviced activities: letting $S_d(x) = \int_0^x H_d^{-1}(1-\alpha)\,d\alpha$,

$$\mathrm{GS}_d = A_d \int_{\tilde{v}_d}^{\infty} v\,dH_d(v) = A_d \int_{1-T_d/A_d}^{1} H_d^{-1}(\alpha)\,d\alpha = A_d S_d(T_d / A_d).$$

The net surplus being the difference between gross surplus and transport cost, in the case of a unique transport cost G_d the total net surplus of consumers is $\mathrm{NS}_d = \mathrm{GS}_d - T_d G_d$.

Fig. a. Demand-supply equilibrium.



2 BINARY CASE

When the activities are located in several destination zones, their microscopic competition has a spatial outreach and gives rise to a macroscopic competition between destination zones.

2A Equilibrium between destination zones

We consider a single origin zone *o* with X_o consumers and two destination zones *d* and *d'*, each of them with its own A_d and H_d. Let us denote by G_d and $G_{d'}$ the transport cost from *o* to *d* and *d'* respectively. The equilibrium state of the economic market is characterized by the endogenous quantities T_d and $T_{d'}$ which count the consumers serviced in *d* and *d'*, and by the endogenous net values $H_d^{-1}(1-\frac{T_d}{A_d})-G_d=F_d$ and $H_{d'}^{-1}(1-\frac{T_{d'}}{A_{d'}})-G_{d'}=F_{d'}$.

Let us assume that the problem is regular, i.e. $A_d \geq 0$, $A_{d'} \geq 0$, $X_o \geq 0$, $X_o \leq A_d + A_{d'}$ and continuity of H_d^{-1} and $H_{d'}^{-1}$ on $]0, 1[$.

The equilibrium state may be analysed with respect to the number of congested zones, in which $T_d = A_d$.

Case A, no congested zone. If one zone, say *d*, is loaded then the maximal net value of vacant activities in *d* is F_d. It must hold that $F_{d'} \leq F_d$ for if $F_{d'} > F_d$ then the last customer of zone *d* would divert to zone *d'*.

Case B, only one congested zone. If one zone, say *d*, is congested then the maximal net value of vacant activities is $F_{d'}$, which satisfies $F_{d'} \leq F_d$ as in Case A.

Case C, both zones are congested. In this case only the minimum value of serviced activities if of economic significance, $\min\{F_d, F_{d'}\}$.

Whatever the case we may define three dual variables ρ_d, $\rho_{d'}$ and u_o as depicted below.

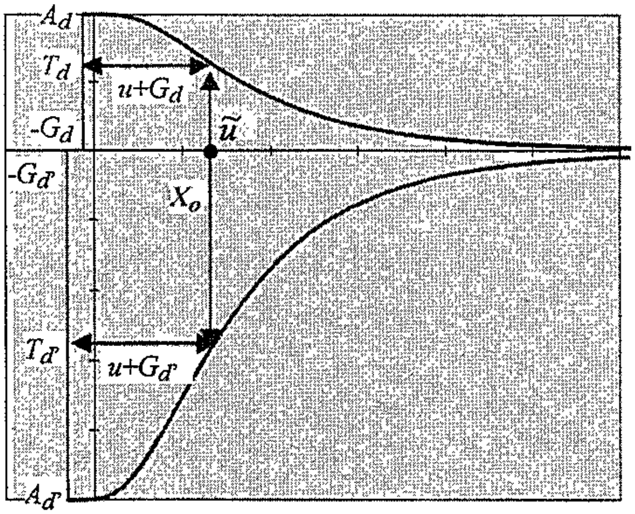
Number of congested zones	Dual variables	Interpretation of u_o
None	$u_o = \max\{F_d, F_{d'}\}$ $\rho_d = 0$ and $\rho_{d'} = 0$	Maximum value of vacant activities
One only, say *d*	$u_o = F_{d'}$ $\rho_d = F_d - u_o$ and $\rho_{d'} = 0$	
Both	$u_o = \min\{F_d, F_{d'}\}$ $\rho_d = F_d - u_o$ and $\rho_{d'} = F_{d'} - u_o$	Minimum value of serviced activities

2B Illustration

When both zones are loaded and uncongested, by replacing $T_{d'}$ with $X_o - T_d$ we obtain the following equation which fixes T_d: $F_d(T_d) = F_{d'}(X_o - T_d) = u_o$. Alternatively, we may relate each volume T_d or $T_{d'}$ to the net value of vacant activities, *u*, by inverting $H_d^{-1}(1 - T_d / A_d) - G_d = u$ into $t_d(u) = A_d(1 - H_d(u + G_d))$.

Figure b represents the competition between two destination zones. The x-axis corresponds to the net value of vacant activities, either for one zone or both. In the upper part (resp. lower part) the y-axis corresponds to the number of serviced consumers T_d (resp. $T_{d'}$). Each volume depends on the net value through $t_d(u) = A_d(1 - H_d(u + G_d))$, which satisfies the conditions $0 \le t_d(u) \le A_d$ and $u = H_d^{-1}(1 - t_d(u)/A_d) - G_d$. Then the equilibrium volumes correspond to the abscissa $\tilde{u}$ at which point $t_d(\tilde{u}) + t_{d'}(\tilde{u}) = X_o$.

Fig. b. Competition between two destination zones.



2C Mathematical study

From the previous discussion, the primal variables T_d and $T_{d'}$ and the dual variables check the following system of conditions which characterizes an equilibrium:

- $T_d - A_d \le 0$, $\rho_d \ge 0$, $\rho_d(T_d - A_d) = 0$,
- $T_{d'} - A_{d'} \le 0$, $\rho_{d'} \ge 0$, $\rho_{d'}(T_{d'} - A_{d'}) = 0$,
- $T_d \ge 0$, $F_d - u_o - \rho_d \le 0$, $T_d(F_d - u_o - \rho_d) = 0$,
- $T_{d'} \ge 0$, $F_{d'} - u_o - \rho_{d'} \le 0$, $T_{d'}(F_{d'} - u_o - \rho_{d'}) = 0$,
- $T_d + T_{d'} - X_o = 0$.

Thus the dual variables may be interpreted as Lagrange multipliers associated with constraints $T_d - A_d \le 0$, $T_{d'} - A_{d'} \le 0$ and $T_d + T_{d'} - X_o = 0$ respectively. Furthermore, let us consider the function $\mathrm{J}(T_d;T_{d'}) = \sum_d A_d S_d(T_d / A_d) - G_d T_d$ (where $S_d(x) = \int_0^x H_d^{-1}(1-\alpha)\,d\alpha$), of which the derivatives are $\frac{\partial J}{\partial T_d} = F_d(T_d)$. The characteristic conditions are the Kuhn-Tucker conditions of the following optimization program:

Th. Optimization program of binary AVA model. An equilibrium state of the binary AVA model is an optimal solution to:

$$\max_{T_d, T_{d'} \ge 0} \mathrm{J}(T_d;T_{d'}), \text{ subject to } T_d - A_d \le 0,\ T_{d'} - A_{d'} \le 0 \text{ and } T_d + T_{d'} - X_o = 0.$$

Proof. The Lagrangian is $L(T_d, T_{d'}, \rho_d, \rho_{d'}, u_o) = J - \sum_d \rho_d (T_d - A_d) - u_o (\sum_d T_d - X_o)$, in which the dual variables ρ_d and $\rho_{d'}$ are non negative. Its derivatives are as follows:

$$\partial L / \partial T_d = F_d - \rho_d - u_o;\ \partial L / \partial \rho_d = A_d - T_d \text{ and } \partial L / \partial u_o = X_o - \sum_d T_d .$$

The Kuhn-Tucker conditions for a saddle point of L are

- $\partial L / \partial T_d \leq 0$, $T_d \geq 0$ et $T_d \partial L / \partial T_d = 0$
- $\partial L / \partial \rho_d \geq 0$, $\rho_d \geq 0$ et $\rho_d \partial L / \partial \rho_d = 0$
- $\partial L / \partial u_o = 0$

i.e. the characteristic conditions for an AVA equilibrium state.

2D Economic interpretation

If zone *d* is loaded and uncongested, i.e. $0 < T_d < A_d$, then at equilibrium the net value of the best vacant activity in zone *d* from zone *o* is $u_o = F_d$, equal to that in zone *d'* if $0 < T_{d'} < A_{d'}$. Thus u_o measures the **accessibility to vacant activities.**

As the objective function J is the sum of the net consumer surpluses in zone *d* and zone *d'*, it measures the **total net consumer surplus.** In the uncongested case, at equilibrium the derivative of the optimal value of J with respect to X_o is equal to u_o.

2E From exponential distribution to gravity model

Assuming exponential distributions $H_d(v) = H_{d'}(v) = 1 - \exp(-\lambda_d (v - m_d))$ for $v \geq m$ or 0 otherwise, we obtain a primitive function $S_d(x) = x[\frac{1 - \ln x}{\lambda_d} + m_d]$ and an objective function $J = \sum_d T_d [\frac{1 - \ln(T_d / A_d)}{\lambda_d} + m_d - G_d]$.

The equilibrium condition of O-D pair *o-d* under $T_d > 0$ is $H_d^{-1}(1 - \frac{T_d}{A_d}) = u_o + G_d + \rho_d$, or equivalently $T_d = A_d \exp[-\lambda_d (G_d + u_o + \rho_d - m_d)]$.

Summing over *d* we obtain a formula $X_o = \sum_d A_d \exp[-\lambda_d (G_d + u_o + \rho_d - m_d)]$ which characterizes the accessibility, and also the optimal value of the objective function, $J^* = u_o X_o + \sum_d T_d (1 / \lambda_d + \rho_d)$.

Assuming identical distributions $H_d = H_{d'}$ and uncongested destinations i.e. $\rho_d = \rho_{d'} = 0$, the formulae reduce to

- $u_o = m + \frac{1}{\lambda} \ln \sum_d \frac{A_d}{X_o} \exp[-\lambda G_d]$ for the accessibility variable,
- $J^* = X_o (u_o + 1 / \lambda)$ for the optimal total net surplus,
- $\frac{T_d}{T_{d'}} = \frac{A_d \exp[-\lambda G_d]}{A_{d'} \exp[-\lambda G_{d'}]}$ for the ratio of two O-D volumes.

Thus the gravity formula is established within a rigorous framework for economic evaluation.

3 GENERAL CASE

3A Basic model

Let us now consider several destination zones, indexed by *d*, and several origin zones, indexed by o. We denote by T_{od} the O-D flow from o to *d*, hence $T_d = \sum_o T_{od}$. The regularity condition becomes $A_d \geq 0$, $X_o \geq 0$, $\sum_o X_o - \sum_d A_d \leq 0$ and the continuity of functions H_d^{-1} on]0, 1[.

Following the same lines as in the binary case, it has been shown (Leurent, 1999) that an activity market equilibrium is characterized by the following system of conditions:

- $T_d \leq A_d$, $\rho_{od} \geq 0$, $\rho_{od}(T_d - A_d) = 0$,
- $T_{od} \geq 0$, $F_{od} - u_o - \rho_d \leq 0$, $T_{od}\ (F_{od} - u_o - \rho_d) = 0$ in which $F_{od} = H_d^{-1}(1 - \frac{T_d}{A_d}) - G_{od}$,
- $\sum_d T_{od} = X_o$.

This makes a generalized complementarity problem, from which an optimization program may be derived under the **last assumption** that activity consumers maximize the net total surplus. Thus two activity consumers located in zone *o* and *o'* and performing activities in zone *d* and *d'*, will exchange their activities if this allows the sum of their net individual values to increase.

Th. Optimization program of general AVA model. An equilibrium state of the general AVA model is a solution to:

$$\max_{T \geq 0} \mathrm{J(T)} = \sum_d A_d S_d(T_d / A_d) - \sum_{od} T_{od} G_{od}$$

$$\text{subject to } \sum_o T_{od} - T_d = 0 \text{ and } T_d - A_d \leq 0 \text{ and } \sum_d T_{od} - X_o = 0.$$

It should be emphasized that a solution to the maximization program also solves the generalized complementarity problem: the last assumption is consistent with all of the previous ones.

Existence and uniqueness corollary. [i] Under the regularity condition, there exists an optimal solution to the general AVA model. [ii] The set of optimal solutions is convex. [iii] The objective function has a unique value at equilibrium.

Proof. [i] because the feasible set is non empty and bounded hence compact, while the objective function J is continuous. [ii] and [iii] stem from the concavity of J.

3B Economic interpretation

The objective function J still represents the total net surplus of the consumers.

The dual variables u_o and ρ_d still contribute as Lagrange multipliers to the Kuhn-Tucker optimality conditions of the maximization program. If $T_{od} > 0$ then $F_{od} - u_o - \rho_d = 0$ which implies that, assuming $T_d < A_d$ and $\rho_d = 0$, $u_o = H_d^{-1}(1 - T_d / A_d) - G_{od}$ is the maximum net value of vacant activities in zone *d* from zone *o*. Conversely if $T_{od} = 0$, either $T_d < A_d$ in which case $H_d^{-1}(1 - T_d / A_d) - G_{od} \leq u_o$ and zone *d* is inferior to other destination zones (from origin *o*), or $T_d = A_d$ in which case $\rho_d = H_d^{-1}(0) - G_{od} - u_o$ measures the difference between the minimum net value serviced in zone *d* from zone *o* and the available value u_o.

3C Extension to congested transport

The dependency of transport costs on the level of traffic may be modelled in AVA by transforming the constants G_{od} into functions of network path flows. Let K_{od} denote the set of network paths from origin *o* to destination *d*, and $K = \bigcup_{od} K_{od}$; the vector of path flows is $\mathrm{f} = (f_k)_{k \in K}$. Each path *k* has its own transport cost function $g_k(\mathrm{f})$.

Then we may express G_{od} as a function of vector f either in a direct way by $G_{od} = \mathrm{g}_{od}(\mathrm{f})$, or via the path costs $G_{od} = \mathrm{h}_{od}\big((g_k(\mathrm{f}))_{k \in Kod}\big)$ in which h_{od} is an additional function (for instance h_{od}=min in minimum cost assignment, or a log-sum formula in logit assignment).

It is straightforward to include the dependency $G_{od}(\mathrm{f})$ into the system of characteristic conditions. However, to avail ourselves of an optimization program, we require the further assumption that these functions should derive from a potential function U, i.e. $G_{od} = \frac{\partial \mathrm{U}}{\partial T_{od}}$. Then the objective function becomes $\mathrm{J(T)} = \sum_d A_d S_d(T_d / A_d) - \mathrm{U(T)}$, or alternatively $J(\mathrm{f}) = \sum_d A_d S_d(T_d / A_d) - \mathrm{U(f)}$ by stating that $T_{od} = \sum_{k \in Kod} f_k$. The last expression arises when link travel time functions $\mathrm{t}_a(x_a)$ are considered, in which $x_a = \sum_{k;a \in k} f_k$: minimum time assignment yields that $G_k(\mathrm{f}) = \sum_{a \in k} t_a$ and $\mathrm{U(f)} = \sum_a \int_0^{x_a(\mathrm{f})} t_a(z)\, dz$.

3D Extension to elastic demand

For every origin zone *o* we may relate the demand volume X_o to the net value of vacant activities, i.e. the multiplier u_o. Let us assume that $X_o = \mathrm{D}_o(\kappa_o - u_o)$, in which D_o is a demand function, κ_o is a constant and $\kappa_o - u_o$ converts the intrinsic value u_o into a price (which is the usual argument of a demand function). The constant κ_o is set together with the function D_o.

We add the following conditions to the previous system of characteristic conditions:

- $X_o \geq 0$, $\mathrm{D}_o^{-1}(X_o) - (\kappa_o - u_o) \leq 0$ and $X_o[\mathrm{D}_o^{-1}(X_o) - (\kappa_o - u_o)] = 0$.

The second inequality states that if no positive X_o corresponds to a given u_o, then $X_o = 0$ is associated to a willingness-to-pay of $\mathrm{D}_o^{-1}(0) \leq \kappa_o - u_o$.

The resulting characteristic system consists in the Kuhn-Tucker optimality conditions of the following maximization program.

Th. Elastic demand AVA model. An equilibrium state (T, X) of the elastic demand AVA model is a solution to:

$$\max_{\mathrm{T,X} \geq 0} \tilde{\mathrm{J}}(\mathrm{T;X}) = \mathrm{J(T)} + \mathrm{J}_{\mathrm{vol}}(\mathrm{X}) = \mathrm{J(T)} + \sum_o \int_0^{X_o} [\mathrm{D}_o^{-1}(x) - \kappa_o]\, \mathrm{d}x$$

$$\text{subject to } \sum_o T_{od} - T_d = 0 \text{ and } T_d - A_d \leq 0 \text{ and } \sum_d T_{od} - X_o = 0.$$

Economic interpretation. Again the objective function $\tilde{J}$ may be interpreted as the total net consumer surplus, measured with respect to alternative consumption (economic goods other than activities). The area under the inverse demand curve, $\int_0^{X_o}[\mathrm{D}_o^{-1}(x)-\kappa_o]\mathrm{d}x$, measures the gross surplus from which the consumers would benefit were they not to consume any activity: then $\int_0^{X_o}[\mathrm{D}_o^{-1}(x)-\kappa_o]\mathrm{d}x-X_o u_o$ measures the net opportunity cost of activity consumption. As the objective function $\tilde{J}$ subtracts the opportunity cost from the net surplus J(T) - $\sum_o X_o u_o$, it represents the net surplus with respect to alternative consumption.

The elastic demand AVA model enables one to simulate the reaction of activity demand to supply measures such as a variation in transport costs.

4 COMPARISON TO OTHER DISTRIBUTION MODELS

4A General discussion

Economic content. Amongst other trip distribution models, only the opportunity model and the accessibility model of Koenig (1974) and Cochrane (1975) represent activities in an explicit way. In the opportunity model there is no value associated with activities, and the destination choice is a random process rather than an economic choice. In the Koenig-Cochrane model (later denoted by ACK), every consumer associates to every activity a subjective value which is a random variable subject to aggregation constraints with respect to first the individual consumer (a given CDF for all gross values) and second the population (same CDF for all consumers). All activities have the same gross mean subjective value, which does not enable one to distinguish one from another (except by location). Lastly, the occupation of activities is associated with destination zone and not with individual activity.

Formal side. The gravity, opportunity, ACK and AVA models possess a mathematical formulation with characteristic conditions; so does a discrete choice model of destination zone. The former models are also endowed with a characteristic optimization program, with nice existence and uniqueness properties. In the single origin case, the optimization program is almost identical for the gravity, ACK and AVA models.

Technical side. The gravity and ACK models may be solved very efficiently by the balancing method or Newton's method. The AVA model may be addressed by Newton's method or standard assignment algorithm (eg. convex combination).

Empiric side. This involves the specification of exogenous variables and the estimation (or inference) of missing exogenous parameters. The AVA model requires as much information as a gravity model or a discrete choice model of destination zone, which is more than required by an opportunity model or the ACK model. As regards estimation or inference, let us only state that AVA can be identified since, in the fixed-demand case with every distribution H_d described by two parameters a_d and b_d, there are O+3D parameters X_o, A_d, a_d and b_d to be estimated using OD observed origin-destination flows.

4B Formal comparison between AVA and ACK

In the ACK model, A_d activities are supplied in each destination zone *d*. These are evaluated on a subjective basis by a given consumer: the subjective gross value of an activity is the outcome of a random variable, with cumulative distribution function *H*.

It is assumed that the market share of zone d is equal to the probability that zone d contains the activity with maximum net value. In the binary case with independent activities and exponential distribution $H(v) = 1-\exp[-\lambda(v-m)]$ for $v \geq m$ or 0 otherwise, this probability amounts to

$$\mathrm{P}_{od} = \frac{A_d \exp(-\lambda G_d)}{A_d \exp(-\lambda G_d) + A_{d'} \exp(-\lambda G_{d'})} = \frac{T_d}{X_o}.$$

When there is no congestion, this is identical to AVA's formula for exponential functions $H_d = H_{d'}$. In ACK the average net subjective value of consumed activities is called the accessibility of zone o and formulated as follows, in which $\gamma \approx 0.5772$ denotes Euler's constant,

$$u'_o = m + \tfrac{1}{\lambda}[\gamma + \ln \textstyle\sum_d A_d \exp(-\lambda G_d)].$$

This looks like AVA's accessibility u_o apart from two differences: first u'_o includes γ, second in u_o the A_d volumes are divided by X_o, which normalizes the volumes. Koenig (1975) also defines the total net surplus to be $\mathrm{J}'^{*} = X_o u'_o$, which is close to AVA's net surplus $\mathrm{J}^{*} = X_o(u_o + 1/\lambda)$.

To model congestion, let us assume that the number of serviced activities in zone d is further constrained by $B_d \leq A_d$. As in ACK the activities cannot be distinguished from each other, the effect is to restrict $T_d = \min\{B_d,\ X_o P_{od}\}$. This amounts to reduce the gross values of zonal activities by ρ'_d such that

$$\frac{T_d}{X_o} = \frac{A_d \exp[-\lambda(\rho'_d + G_d)]}{A_d \exp[-\lambda(\rho'_d + G_d)] + A_{d'} \exp(-\lambda G_{d'})}.$$

If $T_d < B_d$ then $\rho'_d = 0$. If $T_d = B_d$ then $\frac{X_o}{B_d} = 1 + \frac{A_{d'}}{A_d} \exp[-\lambda(G_{d'} - \rho'_d - G_d)]$ hence $\rho'_d = \frac{1}{\lambda}\ln(\frac{A_d}{A_{d'}} \frac{X_o - B_d}{B_d}) + G_{d'} - G_d$ and $A_d \exp(-\lambda(\rho'_d + G_d)) = \frac{A_{d'} B_d}{X_o - B_d} \exp(-\lambda G_{d'})$, yielding that

$$u'_o = m + \tfrac{1}{\lambda}[\gamma + \ln \textstyle\sum_d A_d \exp(-\lambda[G_d + \rho_d])] = m - G_{d'} - \tfrac{1}{\lambda}[\ln(\frac{X_o - B_d}{X_o A_d}) - \gamma].$$

Koenig (1975) still considers a total net surplus of $\mathrm{J}'^{*} = X_o u'_o$.

In AVA it is necessary to identify the values of the B_d remaining activities. A natural assumption is that they correspond to the upper part of the distribution. Then the only adaptation is to replace $(A_d,\ H_d)$ with $(B_d,\ H'_d)$ such that $H'_d(v) = 0$ if $H_d(v) < 1 - \frac{B_d}{A_d}$ or $H'_d(v) = \frac{A_d}{B_d}[H_d(v) - (1 - \frac{B_d}{A_d})] = 1 - \frac{A_d}{B_d}[1 - H_d(v)]$ if $H_d(v) \geq 1 - \frac{B_d}{A_d}$, hence

$$H'^{-1}_d(\alpha) = H^{-1}_d(1 - (1-\alpha)\tfrac{B_d}{A_d}).$$

It may be easier to maintain A_d and H_d in the objective function and to replace constraint $T_d \leq A_d$ with $T_d \leq B_d$. Assuming an exponential function H_d, the equilibrium conditions are $T_d = A_d \exp[-\lambda_d(G_d + u_o + \rho_d - m_d)]$ and $T_d = \min\{B_d, A_d \exp[-\lambda_d(G_d + u_o - m_d)]\}$.

If $T_d < B_d$ then $\rho_d = 0$. If $T_d = B_d$ then $u_o + \rho_d = \frac{1}{\lambda_d}\ln(\frac{A_d}{B_d}) + m_d - G_d$ which is injected into $X_o = B_d + A_{d'}\exp[-\lambda_{d'}(G_{d'} + u_o - m_{d'})]$ to yield that

$$u_o = m_{d'} - G_{d'} - \frac{1}{\lambda_{d'}}\ln(\frac{X_o - B_d}{A_{d'}}).$$

Thus $\rho_d = \frac{1}{\lambda_d}\ln(\frac{A_d}{B_d}) + \frac{1}{\lambda_{d'}}\ln(\frac{X_o - B_d}{A_{d'}}) + m_d - m_{d'} + G_{d'} - G_d$. Assuming identical functions $H_d = H_{d'}$, the formulae of AVA's ρ_d and ACK's ρ'_d are identical. However AVA's surplus $J^* = X_o(u_o + 1/\lambda) + B_d\rho_d$ differs notably from Koenig's formula for J'^*.

4C Numerical illustration

We shall apply the ACK and AVA models to a binary case, either with congestion or not.

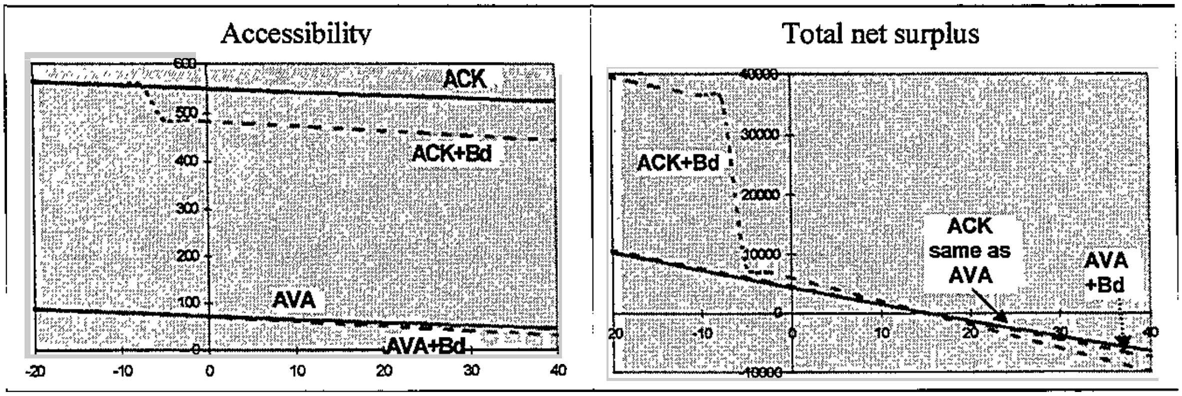
Data. As regards the two destination zones, we assume identical, exponential distributions H_d with parameters m = 32.7 and $1/\lambda$ = 73.0 (in euros) which mimic the distribution of wages in France for one working day. Supplied volumes are $A_d = 200$ and $A_{d'} = 500$. In the congested case, it is assumed that in zone d the number of serviced customers satisfies $T_d \leq B_d = 100$. This additional constraint is interpreted in AVA as the restriction of distribution (A_d, H_d) to its B_d higher values.

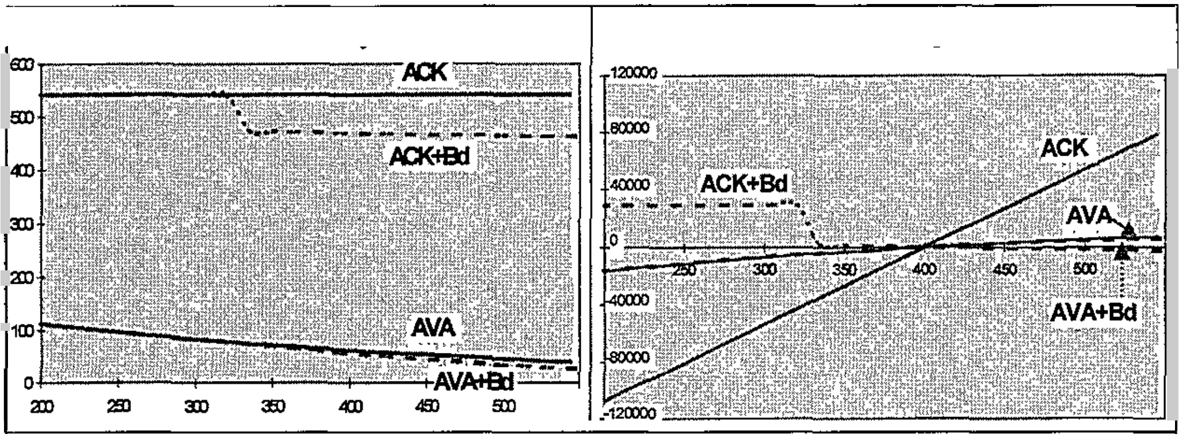
As regards the origin zone, a reference situation is defined with demand volume $X_o = 400$, transport costs $G_d = 6$ and $G_{d'} = 15$ euros.

Tests. We shall conduct two series of tests, **first** by varying $G_{d'}$ and second by varying X_o. In each case four models were applied: ACK without B_d, ACK with B_d, AVA without B_d and AVA with B_d. The results of each application consist in the accessibility u_o and the total net consumer surplus, measured with respect to the reference situation.

Variation of $G_{d'}$. Without congestion the total net surpluses of AVA and ACK are identical (fig. c); the accessibilities are identical up to a constant term. Congestion changes AVA's results to a limited extent, but its effects on ACK are important because Koenig's formula $J^* = X_o u_o$ (wrongly) omits the contribution of the ρ_d variables.

Fig. c. Variation wi respect to $G_{d'}$.





4D Summary

Despite the word analogy, the economic interpretations of the AVA and ACK models are quite different. This is reflected by the mathematical formulae. In particular the formula for the accessibility in the binary model contains an obvious difference: the supplied quantities A_d are divided by the demand volume X_o in AVA, not in ACK. Then ACK induces a bias towards large size, which is wrong according to AVA's assumptions.

Let us also remark that zone-specific distributions of supplied activities (A_d, H_d) may be identified in AVA, which may be used to represent structural differences across destination zones (eg. center versus periphery in an urban area).

5 CONCLUSION

The AVA model of accessibility to vacant activities is founded on explicit microeconomic assumptions: each activity has individual objective economic value, occupation and location. Each activity consumer, from his origin zone, derives from the consumption of an activity a net value equal to the activity gross value minus the transport cost. A marginal consumer will choose the vacant activity with maximal net value. At market equilibrium, in each destination zone only the best activities are consumed, each of them by a single consumer.

Thus the concepts of surplus and accessibility have a clear and rigorous significance; they are characterized by quantitative variables which stem from the supply-demand equilibrium on the activity market.

By representing the supplied activities of each destination zone in a probabilistic way, we have endowed the AVA model with a standard mathematical formula: the model is characterized by a generalized complementarity problem as well as by a concave maximization program, of which the objective function represents the total net consumer surplus. The accessibility from an origin zone is the dual variable associated to the split of the zonal demand volume between the destination zones.

Our mathematical formulation enables one to apply AVA in the same way as previous distribution models, to which it is superior due to its microeconomic foundation.

Further work on AVA should address the differentiation of the consumers located in a given origin zone, and their selection by the activity suppliers. This would lead to an explicit representation of activity supply and its economic surplus.

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