A GENETIC ALGORITHMS BASED APPROACH FOR SOLVING OPTIMAL TOLL LOCATION PROBLEMS

A.Sumalee and S.P.Shepherd
Institute for Transport Studies, University of Leeds, UK.
asumalee@its.leeds.ac.uk; sshepher@its.leeds.ac.uk

1. INTRODUCTION

The concept of road pricing is a long-established fiscal mechanism aimed to control the demand of road usage (Pigou, 1920; Knight, 1924; and Walters, 1961). This policy emerged from the belief that car users have not paid their full costs of road uses (Vickrey, 1969). The idea is proposed that by alleviating the appropriate tolls the traffic level and distribution will organise itself in an optimal way. The *"optimal way"* is regularly referred to the traffic pattern and demand level that maximise the social welfare (Yang and Huang, 1998 and Verhoef, 2000). Under the assumption that all links in a network can be tolled, the first-best optimal toll can be found easily by solving the system optimal traffic assignment problem (Sheffi, 1985). However, the problem becomes much more complex when only a subset of links is to be tolled (called the second-best optimal toll problem). Since the emergence of the concept of the second-best optimal toll problem, many economists have illustrated the approach to quantify the optimal toll level. (Levy-Lambert, 1968; Marchand, 1968; Vickrey, 1968; Arnotte et al, 1990; McDonald, 1995; Liu and McDonald, 1999; Verhoef 2000; Small and Yan, 2001).

In the optimisation context, the second-best marginal cost toll problem is categorised as a mathematical programming problem with equilibrium constraints (MPEC), a special case of the bi-level optimisation programming problem (BLPP). The regulator tries to set the toll locations and toll levels to optimise his or her objective whilst the users attempt to minimise their own travel costs. The optimal toll problem can also be seen as a special case of the network design problem. Several methods have been proposed to tackle this challenging problem. Most procedures to solve the BLPP use derivatives and can be put into one of the following categories, heuristic iterative optimisation method (Steenbrink, 1974; Allsop, 1974; Suwansirikul et al; 1987), transforming the BLPP to a single level optimisation program, linearisation method (LeBlanc and Boyce, 1986; and Ben-Ayed et al, 1988), and stochastic search methods[1] (Friesz et al, 1992; Cree et al, 1998; and May et al, 2002). There exists a diverse range of techniques used to transform the BLPP to a single level optimisation program. These include sensitivity based analysis (Friesz et al, 1990;Yang, 1997;

[1] e.g. simulated annealing and genetic algorithms

Yang and Bell, 1997), Karush-Kuhn-Tucker (KKT) based method (Marcotte, 1983; Marcotte, 1986; and Verhoef, 2000), using the system optimal solution to formulate the set of tolls for the second-best case under user equilibrium (Bergendorff et al, 1996; Hearn and Ramana, 1998; and Hearn and Yildirim, 2000), and recently a marginal function based method (Meng et al, 2001).

In the real world, the design of pricing schemes not only involves determining the toll levels but also the locations of the toll points. The common practice of various real-world cases or on-desk studies is to apply the judgmental approach or a trial and error process to seek the optimal toll location and the toll level (Shepherd et al, 2001; May et al, 2002; and Sumalee, 2001). The "trial and error process" normally starts by defining a set of possible charging cordons and their associated common toll levels. Then, each option will be tested with traffic modelling software, such as SATURN (Van Vliet et al, 1982). The benefit of each scheme will be calculated from the modelling output and the best scheme will be chosen. The sub-optimality of this judgmental design is well addressed in a recent study (May et al, 2002). Recently, there have been increasing attempts to develop the analytical approach to tackle the optimal toll location problem (Mun et al, 2001; Shepherd et al, 2001; Verhoef, 2000; Hyman and Meyhew, 2002; Yang et al, 2002; and Shepherd and Sumalee, 2002).

This paper firstly demonstrates a derivative based method proposed by Verhoef (2000) to solve the optimal toll level and location problems (named CORDON and LOCATE respectively). New methods are then developed to tackle the problems based on genetic algorithms (GA) and parallel genetic algorithms (PGA). The paper consists of four further sections. The next section shows the formulation of the optimal toll problem as an MPEC and explains briefly the derivative based method. Section three then proposes four new methods based on GA and PGA including GA-CHARGE, GA-LOCATE, GA-LOCATEII, and PGA-ALL. Section four displays numerical results comparing the derivative, GA, and PGA based approaches. The final section draws conclusions and discusses the future research.

2. DERIVATIVE BASED METHOD TO OPTIMAL TOLL DESIGN

2.1 Optimal toll level on a specified set of links

The problem of defining the optimal toll level on a specified set of tolled links (termed OPT1) can be formulated as a MPEC. Verhoef (2000) proposed the method to solve the OPT1 by maximising the following Lagrangian[2]:

[2] Notation is given in appendix A.

$$\Lambda = \sum_i \int_0^{T_i} D_i(x)dx - \sum_j \sum_p d_{jp} \cdot F_p \cdot c_j + \sum_p l_p \cdot \left(\sum_j d_{jp} \cdot \left(d_{jp} \cdot \left(c_j + e_j \cdot f_j \right) - D_i \ \right) \right) \quad (1)$$

subject to the set of feasible path flows and non-negative path flows. The first two terms represent the Marshallian measure of social welfare which the leader tries to maximise. The l_p are the Lagrange multipliers associated with each used path under the equilibrium condition. The third term associated with the Lagrange multiplier is the complementarity slackness constraint for the user equilibrium condition (Smith, 1979). Note that only the used paths under equilibrium condition ($F_p > 0$) are included in the Lagrangian in order to reduce the complementarity slackness condition to the normal equality constraint. Verhoef (2000) derived the first order condition of this Lagrangian. Shepherd et al (2001a) utilised this first order condition to develop the mathematical program, termed CORDON, linked with SATURN (Van Vliet et al, 1982) to solve OPT1.

2.2 Optimal toll location in a general network

The other problem when designing a toll based road pricing system is to define the optimal toll point locations given the desired number of tolled points (termed OPT2). A related problem is to find the optimal number of tolled points and locations simultaneously while considering implementation costs (termed OPT3). The most straightforward approach to solve the optimal toll location problem is to test all the combinations of tolled points. However, this would be computationally demanding due to the massive number of possible combinations of tolled points. Suppose that we try to choose the best *t* tolled links from the *j* links in the network, the number of all possible combinations of the tolled links will be $\frac{j!}{t!(j-t)!}$. Verhoef (2000) proposed an incremental approach which Shepherd et al (2001b) adapted and termed LOCATE.

The LOCATE process is an extension of CORDON and involves building up a list of toll points incrementally, by choosing links one by one on the basis of a location index. The location indices are the approximation of the welfare gains that would result from placing optimal charges in particular locations. They use the predicted toll from the first iteration of the CORDON process combined with the shadow prices associated with the link(s) considered. Although previously selected toll points are always included, the charge levels are allowed to vary each time an additional link is added. Those interested in the details of the CORDON process and the LOCATE process should refer to Shepherd et al (2001a) and Shepherd et al (2000b) respectively.

Shepherd et al (2001b) already showed that LOCATE can fail to identify the best pair of tolled links from a simple five-link network. This happened because the best “single” tolled link was not part of the best “pair”. To overcome this weakness Verhoef (2000) suggested a greedy search could be used. However, it is not practical to implement this strategy with large-scale networks due to the number of possible combinations mentioned earlier. Thus, the idea of genetic algorithms (GA) is adopted to generate combinations of tolled points instead. Also, an alternative approach based on parallel genetic algorithms (PGA) is developed to simultaneously solve the optimal toll level and location problems without using the location indices. The next sections introduce the concept of GA and PGA based methods to solve the problems OPT1, OPT2, and OPT3.

3. GENETIC ALGORITHM BASED METHODS

3.1 Introduction to genetic algorithms

Genetic algorithms (GA) are one of the artificial intelligence exhaustive searching techniques; they are stochastic algorithms whose search methods model some natural phenomena: genetic inheritance and Darwinian strife for survival. Davis and Steenstrup (1987) stated that:

“The metaphor underlying genetic algorithms is that of natural evolution. In evolution, the problem each species faces is one of searching for beneficial adaptations to a complicated and changing environment .The ‘knowledge’ that each species has gained is embodied in the makeup of the chromosome of its members.”

The basic idea of the GA approach is to code the decision variables of the problem as a finite string (called ‘chromosome’) and calculate the fitness (objective function) of each string. Chromosomes with a high fitness level have a higher probability of survival. The surviving chromosomes then reproduce and form the chromosomes for the next generation through the ‘crossover’ and ‘mutation’ process. The method of GA is widely applied in many disciplines, but most applications have to modify the GA to the problem or change the problem to be compatible with GA. The main parts in the modification process are the design of chromosome encoding and of the genetic operators (crossover and mutation processes) in order to maintain the search within the feasible space. In the following sections, four methods are developed to solve the OPT1, OPT2 and OPT3 problems.

3.2 A method to solve optimal toll levels (GA-CHARGE)

The GA-CHARGE approach is developed to solve the OPT1 problem. The process of GA-CHARGE randomly generates an initial set of chromosomes representing possible combinations of charge levels on a predefined set of links. The benefits in terms of social welfare improvement are evaluated for each charge level by running SATURN. GA-CHARGE then selects the parent chromosomes for the next generation based on the performance of each chromosome. Since the fitness value in GA-CHARGE can be negative, the selection is based on the tournament selection process (Michalewicz, 1992). The genetic operators, crossover and mutation, are then randomly applied to the parents to produce the offspring.

Chromosome encoding

Let t be the number of predefined tolled links and let r be the predefined maximum toll level. Each chromosome represents a set of toll levels for the t-tolled links in binary format. The structure of the chromosome is therefore a matrix **A** with t columns and k rows where k is determined by the number of digits required to represent the maximum toll in binary format. Figure 1 shows an example chromosome (**A** matrix) for ten tolled links. The toll on each link is defined by the binary number in each column which is shown in the bottom row.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$
$$\begin{matrix} 31 & 21 & 3 & 16 & 11 & 20 & 15 & 10 & 19 & 22 \end{matrix}$$

Figure 1: Chromosome structure for GA-CHARGE

Crossover and mutation process

The crossover process is designed to randomly choose the blocks from two “mated” chromosomes and switch the values in the blocks (see Figure 2).

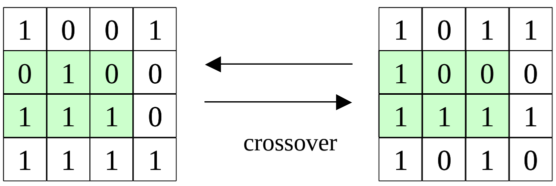

Figure 2: An example of the crossover process in GA-CHARGE

After the crossover process, the mutation process is applied to the offspring. The mutation process randomly chooses cells to be “mutated”. If selected, the value in that cell is changed from 0 to 1 or vice-versa.

3.3 A method to solve optimal toll location based on location indices (GA-LOCATE)

This section explains the approach to use GA to solve the optimal toll location problem, termed GA-LOCATE. The GA process is used to randomly generate and evolve the combinations of the tolled points (chromosome). The location index (see Section 2.2) of each combination is calculated and used as its fitness value. The selection process is based on “stochastic universal” which uses a single wheel spin. The so-called “roulette wheel” is constructed where each slot represents a chromosome. The slots are sized according to the fitness of each chromosome. The size then represents the probability of a chromosome being selected.

Chromosome encoding

The user inputs the number of tolled points required. The adapted chromosome for OPT2 varies the length of chromosome to represent the required number of tolled links and each bit represents a selected link. A list of candidate links can be prepared in advance to reduce the problem. With this structure, the length of the chromosome already controls the required number of tolled points. However, one problem with this structure is duplicating the selected links during the genetic operations.

Crossover and mutation process

Two points on the chromosome are randomly chosen and all “bits” in between these two points of the parents are switched. The mutation process is normally used to avoid the premature convergence of the GA process. In the traditional GA process, the value assigned to each bit is either 1 or 0 (as used in GA-CHARGE). In our modified chromosome, the possible value in each bit is the integer number between 1 and the highest number of candidate links.

3.4 A GA based method to solve toll location problems with implementation costs (GALOCATEII)

The GA-LOCATEII process is developed to solve the problem OPT3. The general algorithm is similar to GA-LOCATE but includes implementation costs. The traditional chromosome structure of GA, string of binary bit, is adopted since there is no constraint on the number of toll points required in this problem. One column is required for each candidate link. If the value is 1, the corresponding link is to be tolled. The location index is calculated and the implementation and operation costs per toll point are subtracted. The standard crossover and mutation processes are also adopted. Note that since the fitness value (location indices net of costs) can be negative the linear ranking proposed by Whitley (1989) linked with stochastic universal sampling is adopted. The slots in the roulette wheel are sized according to the chromosome at rank *i*, where the first is the best chromosome, by the following equation:-

$$p_i = \frac{1}{\|P\|} \cdot \left(2 - c + (2c - 2) \cdot \left(\frac{\|P\| - i}{\|P\| - 1} \right) \right) \qquad (2)$$

where $\|P\|$ is the size of the population set *P*, and $1 \le c \le 2$ is *"the selection bias"*: higher values of *c* cause the system to focus more on selecting only the better individuals. The best individual in the population is thus selected with the probability $\frac{c}{\|P\|}$; the worst individual is selected with the probability $\frac{2-c}{\|P\|}$.

3.5 A parallel genetic algorithms based method to solve OPT3 (PGA-ALL)

The process of GA-LOCATEII described in the previous section relies on the location indices which is only an approximation of the benefits. In order to avoid using an approximation of the benefits, the method based on *"Parallel Genetic Algorithms"* (PGA) is developed *to solve the optimal toll location and optimal toll level simultaneously* (named PGA-ALL). The parallel approach is adopted to reduce the computational workload in GA. The underlying idea of PGA is to decompose the feasible search space of the traditional GA into a number of disjoint partitions with or without *"communication"* between them.

For the optimal toll location problem, each partition is comprised of a set of candidate links. The framework of PGA adopted in this paper is the master and slave nodes framework. In this framework, the sets of subpopulation are created and assigned to slave nodes. Independent populations are associated to nodes, called local population (P_L), see Figure 3. In our OPT3 problem, a set of candidate links is assigned to each slave node. The normal GA process is applied to each slave node in parallel to evolve the chromosomes in each local population (in our case, the optimal combination of tolled links and their optimal toll levels). The PGA will then search for the optimal combination of tolled links in each partition separately with the *“migration”* of the strongest chromosome to other partitions during the search.

At the end of each generation, each slave node sends the strongest chromosome in their local population to the master node. The master node, representing the global communication between slave nodes, then chooses the best global chromosome and sends it back to all slave nodes. Each independent slave node replaces the worst local chromosome by the new best global chromosome received.

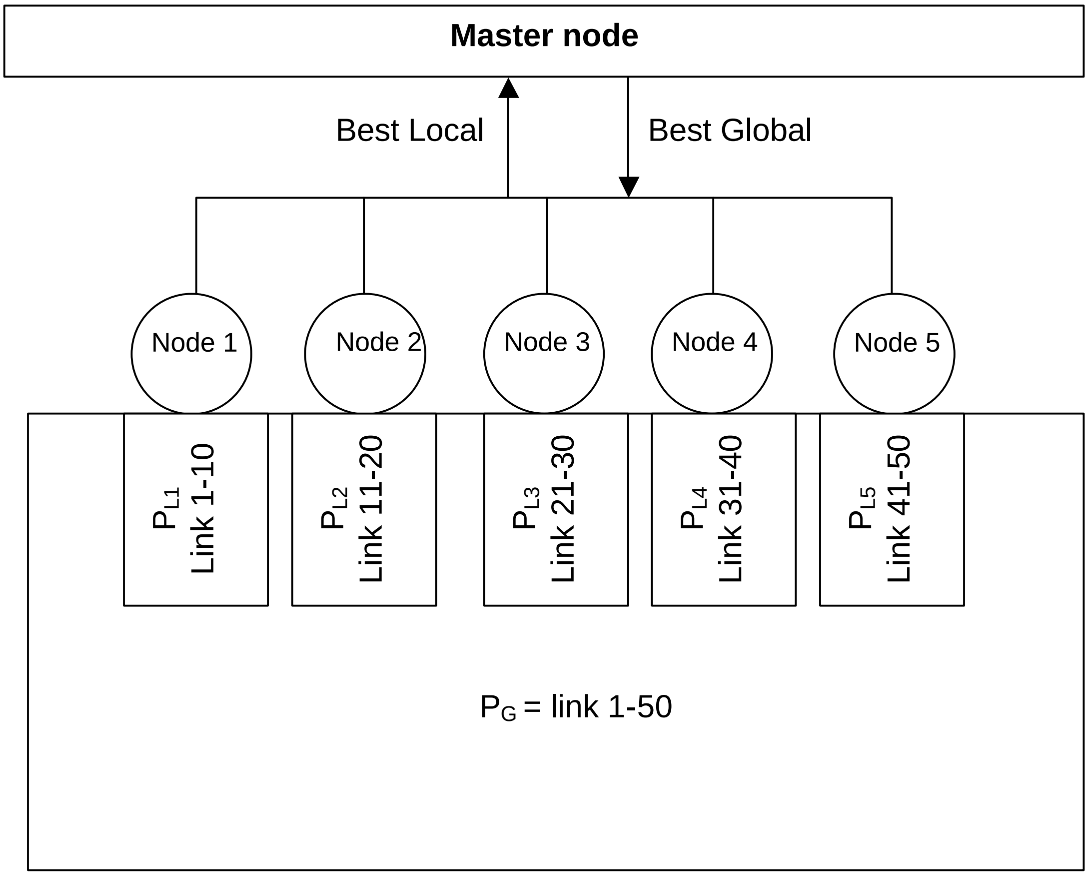

Figure 3: Master-Slave nodes framework for parallel genetic algorithms

In PGA-ALL, the normal GA process applied to each slave node uses the selection, crossover, and mutation process similar to those explained earlier.

4. NUMERICAL RESULTS

4.1 Network description and experimental setting

In this section, the methods developed in the previous sections are tested with a medium-scale network. Figure 4 shows the network which is based loosely on the City of Leeds network in the UK. It should be noted, as the detail of the network has been reduced to decrease the complexity and computation time, the network cannot be considered as a comparative model to the real Leeds network. There are 89 directed links and 14 zones in this network. The triangular nodes represent the zones. The network is a bufferised version of a SATURN network, which means the supply is represented by independent flow-delay relationships for each link. The network is used for the following tests:-

i. Given three pre-defined charging cordons, the CORDON and GA-CHARGE processes are used to find the optimal toll levels around each cordon, see Figure 4 (OPT1);
ii. Given the desired number of tolled links, the LOCATE and GA-LOCATE processes are applied to find the optimal location of the tolled links (OPT2) and charge levels are then optimised using CORDON;
iii. Finally, GA-LOCATEII and PGA-ALL are applied to find the optimal number of tolled points, their locations and toll levels assuming implementation costs (OPT3).

4.2 Numerical results

The results of OPT1

Figure 4 shows the three predefined charging cordons, i.e. inner, intermediate, and outer cordons. The CORDON and GA-CHARGE processes are employed to find the optimal toll on each toll point of these three cordons. Tables 1, 2, and 3 show the optimal toll levels and benefits found by CORDON and GA-CHARGE for the inner, intermediate, and outer cordons respectively. The percentage in brackets is the relative social welfare improvement compared to the first-best condition.

The first-best condition is to apply the marginal cost tolls derived from the system-optimum assignment on all links. In our case, the social welfare improvement for the first-best condition is £5,213 per single AM peak period. The optimal uniform tolls around each cordon are also calculated using a standard univariate optimisation method. The optimal uniform tolls for the inner, intermediate, and outer cordons are

£0.21, £0.19, and £1.04 with social welfare improvements of £166, £445, and £923 per single AM peak period respectively.

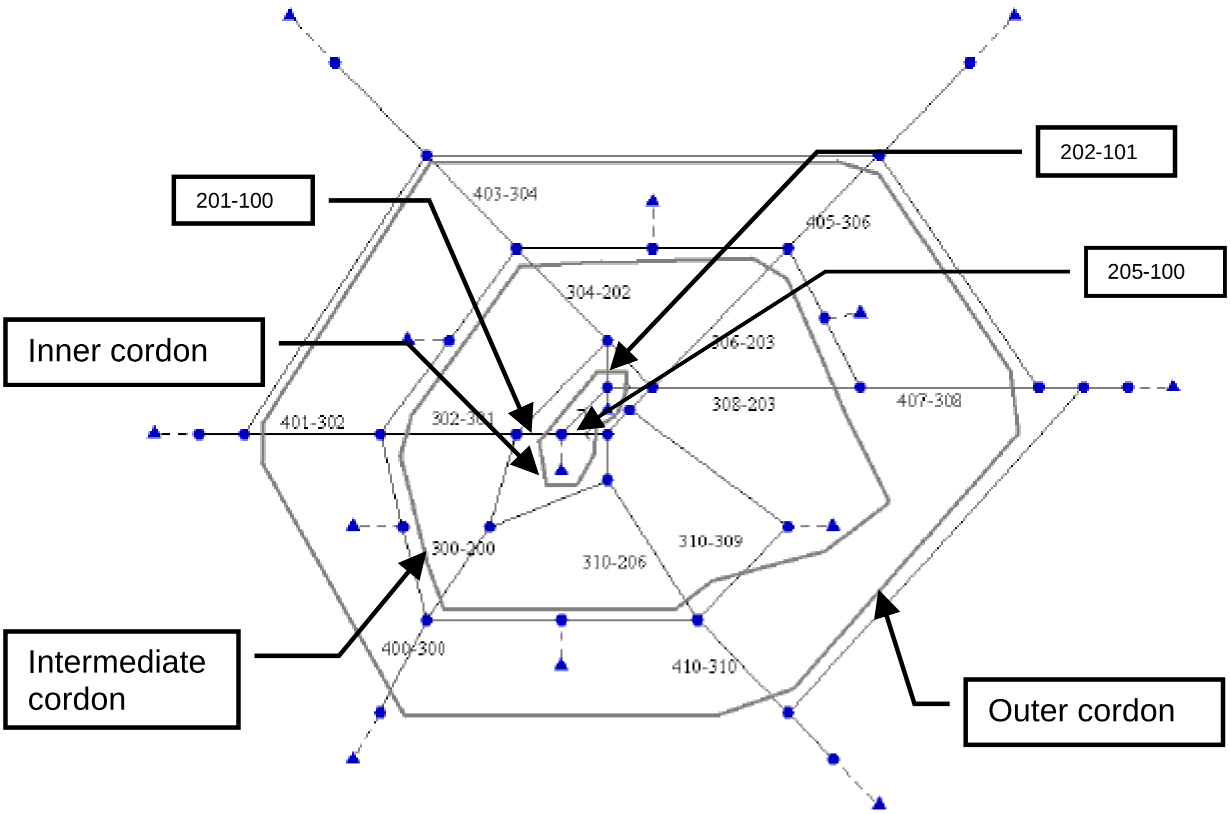

Figure 4: MINILEEDS network used in the numerical tests

From the tables note that allowing the tolls to vary around the cordons increases the benefits significantly. Furthermore applying GA-CHARGE gives higher benefits than the solution produced by CORDON in all cases[3]. The benefits increase by 23%, 8%, and 12% for the inner, intermediate, and outer cordons respectively. Figure 5 illustrates the process of GA-CHARGE for the outer cordon which consists of 6 tolled links. The Y-Axis is the fitness value of each chromosome which is the value of social welfare improvement (£ per single AM peak). The X-Axis is the chromosome number. Note that in this test, the population size is 30 with 50 generations.

[3] Note that the CORDON process did not converge properly for the cordons due to subtle changes in the path sets and so the benefits are not necessarily optimal – hence there is room for improvement which is where GA-CHARGE can obtain extra benefits.

INNER CORDON		
Link	Optimal toll from CORDON (£)	Optimal toll from GA-CHARGE (£)
201-100	0.75	0.68
202-101	1.07	0.90
205-100	0.41	0.24
Social welfare improvement (£ per single AM peak)	305 (5.8%)	375 (7.2%)

Table 1: Optimal tolls and benefits for the inner cordon in MINILEEDS network calculated by CORDON and GA-CHARGE

INTERMEDIATE CORDON		
Link	Optimal toll from CORDON (£)	Optimal toll from GA-CHARGE (£)
302-201	0.38	0.34
304-202	0.62	0.63
306-203	0.66	0.73
308-203	0.73	0.73
310-309	0.09	0.09
310-206	0.07	0.08
300-200	0.09	0.09
Social welfare improvement (£ per single AM peak)	1,005 (19.2%)	1,084 (20.8%)

Table 2: Optimal tolls and benefits for the intermediate cordon in MINILEEDS network calculated by CORDON and GA-CHARGE

OUTER CORDON		
Link	Optimal toll from CORDON (£)	Optimal toll from GA-CHARGE (£)
401-302	0.75	0.83
403-304	1.13	1.40
405-306	1.17	2.05
407-308	1.03	1.39
410-310	1.02	1.07
400-300	0.30	0.65
Social welfare improvement (£ per single AM peak)	1,166 (22.3%)	1,305 (25%)

Table 3: Optimal tolls and benefits for the outer cordon in MINILEEDS network calculated by CORDON and GA-CHARGE

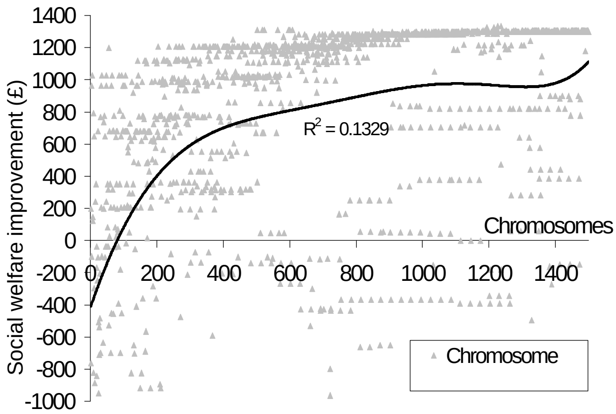

Figure 5: The GA-CHARGE process and the trend of improvement in the fitness value

This means GA only sampled 1,500 chromosomes from all possible combinations. According to the chromosome encoding of GA-CHARGE presented earlier, the total number of possible chromosomes is 2^{t^k} . In this case, t is equal to 6 tolled links and k is equal to 10, the number of digits required in binary format to represent the maximum possible toll given as 1000 seconds, resulting in 2^{600} possible combinations. Note the extremely small ratio between the sampled chromosomes and the possible combinations.

The results of OPT2

The problem of OPT2 is to identify the optimal location of tolled links given the desired number of tolled points. The MINILEEDS network is used again in this test. Two tests are conducted, finding the best six and best ten optimal tolled links. LOCATE and GA-LOCATE are applied to the problems. The total number of directed links in the network is 89. Thus, the possible number of combinations for the problem of six and ten optimal tolled links are approximately 5.8×10^8 and 5.0×10^{12} respectively, which is impractical to implement through enumeration or greedy search methods.

Figure 6 is used to explain the results obtained from LOCATE and GA-LOCATE[4]. The arrows in the figure represent the links selected. From Figure 6, LOCATE selected links A, B, C, D, E, and F as the best six tolled links. Then, LOCATE added links G, H, I, and J as the additional four links for the best ten tolled links. On the other hand, GA-LOCATE selected links A, B, C, D, E, and L as the best six tolled links. Note that GA-LOCATE only picked one different link compared to the set of best six tolled links selected by LOCATE (link L rather than link F). GA-LOCATE then selected links A, B, C, D, E, G, I, K, M, and N as the best ten tolled links. Seven links out of ten links selected by LOCATE are also selected by GA-LOCATE.

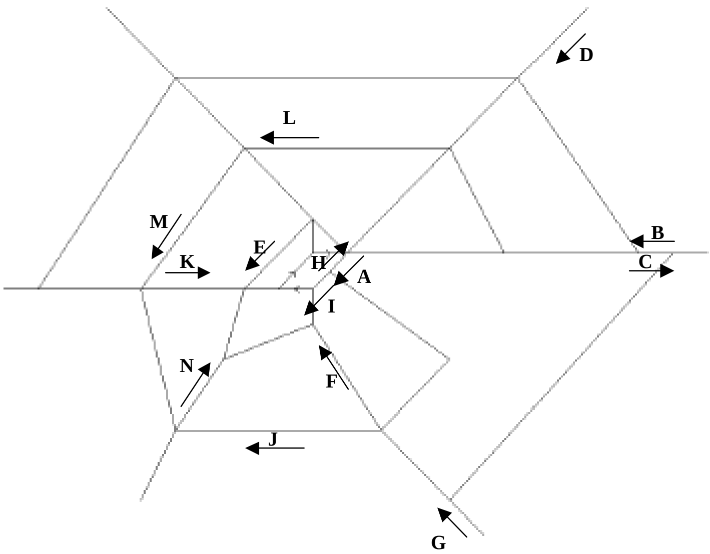

Figure 6: Location of the best 6 and 10 tolled links from LOCATE and GA-LOCATE

Table 4 shows the social welfare improvement in pounds per single peak hour (the percentage in the bracket is the relative welfare improvement compared to the first-best condition). The optimal benefit of the best six and ten tolled links chosen by GA-LOCATE is only slightly higher than those from LOCATE (approximately 0.9% and 2.9% respectively). The difference is that LOCATE has to include all previously

[4] Links B and C are the same link but in the opposite direction. Also, links A and H are the same link but in the opposite direction.

selected links within its solution whereas GA-LOCATE can drop links e.g. link L is not included in the best 10 links even though it is included in the best 6 link solution. It should be noted that even though the LOCATE and GA-LOCATE methods rely on indices which could contain errors, both methods produce solutions which give rise to 85% of the first best conditions even with only 6 toll points. The fact that adding a further 4 links only gives a marginal increase in benefits suggests that the optimal number of toll points when considering implementation costs would be somewhere between 5 and 10 links.

Method	Benefit for best 6 tolled links	Benefit for best 10 tolled links
LOCATE	£4,385 (84.1%)	£4,611 (88.4%)
GA-LOCATE	£4,427 (86.8%)	£4,745 (91%)

Table 4: Benefits of the best 6 and 10 tolled links from LOCATE and GA-LOCATE

The results of OPT3

In this test, the implementation and operation costs per toll point are calculated using a discounted value over a 30-year period. The cost per toll point is assumed to be £100 per toll point per peak-hour based on estimates by Oscar Faber (2001). GA-LOCATEII is used to find the optimal number of tolled points and their locations. GA-LOCATEII identified 10 as the optimal number of tolled links. It selected the best ten tolled links that were chosen by GA-LOCATE previously. This result is wrong since the net benefit for the best 6 tolled links is £3,827 per single peak hour which is actually higher than that from the net benefit from the best 10 links (£3,745 per single peak hour).

The gross location indices for the best 6 and 10 tolled links are £6,465 and £12,192 per single peak hour and the indices net of costs are £5,865 and £11,192. Thus, even after subtracting the costs from the location indices, the set of 10 tolled links remains better than the set of 6 tolled links. The location indices are overestimated for both sets of tolled links. Experience suggests that the toll predictions used in the location index are always an over-estimate of the true optimal tolls i.e. the error terms associated with a link are always positive[5]. Thus we suggest that as the number of links considered is increased then the error in the location index is increased. This does not cause any difficulties for OPT2 as the number of links considered is constant and implementation costs are equal. However as seen here the OPT3 problem has a variable implementation cost and as the magnitude of the errors vary

[5] This is our experience with the CORDON process though we have not yet been able to prove this in a general case.

with links considered the solution selects the wrong combination. Further research is required to improve the performance of the location index approach.

PGA-ALL is also applied to the OPT3 problem. PGA-ALL selects 4 links shown in Figure 7 as the optimal combination of tolled links. The number beside each link is the optimal toll also found by PGA-ALL. Table 5 compares the net benefits

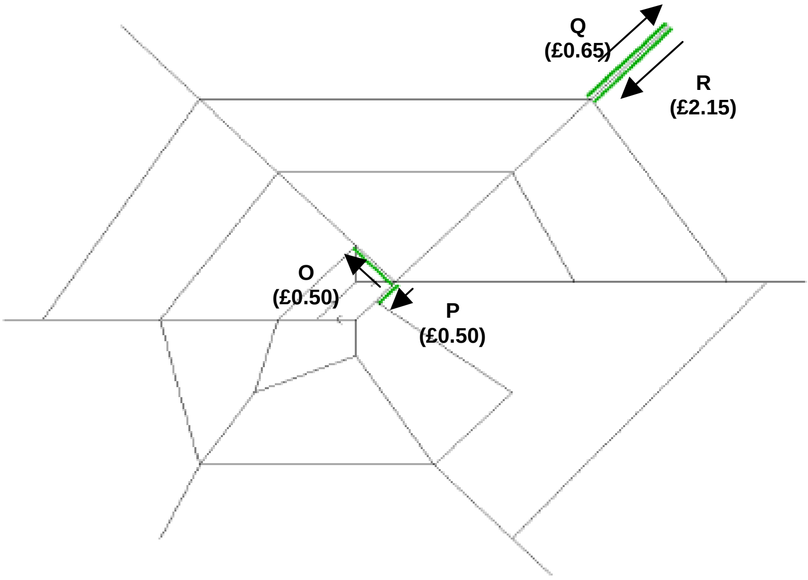

Figure 7: Optimal toll locations selected by PGA-ALL

Methods	No. of tolled links selected	Net benefits
GA-LOCATE	6	£3,720
GA-LOCATEII	10	£3,570
PGA-ALL	4	£2,863

Table 5: Comparison of the net benefits of the best tolled links found by GA-LOCATE, GA-LOCATEII, and PGA-ALL

The point that should be made is that the toll levels used in PGA-ALL are discrete toll levels (with an increment of 15 pence) compared to the continuous toll levels used in GA-LOCATE and GA-LOCATEII. Thus, the benefits of the tolled links found by GA-LOCATE and GA-LOCATEII with the discrete toll levels are used in this table instead. It should be noted that the number of the optimal tolled points used in GA-LOCATE is identified by the user. This is done during the incremental process of LOCATE. When a new link is added to the toll combination in LOCATE, the increase in the benefit is considered and it was noticed that the cost of adding the seventh tolled link is actually higher than the increases benefit. Thus, the optimal number of tolled links should be six. Then, GA-LOCATE is used to identify the best combination of six tolled links.

From Table 5, it seems that GA-LOCATE with user intervention is the best approach compared to GA-LOCATEII and PGA-ALL. The benefits of the tolled links selected by GA-LOCATE is higher than those from PGA-ALL about 20 %. GA-LOCATEII also outperformed PGA-ALL in terms of the net benefits. From the results shown in Figure 6 earlier, PGA-ALL selected two new links, link O and link Q which do not appear in the results of GA-LOCATE and GA-LOCATEII.

The main reason for the failure of PGA-ALL is the process between Master and Slave nodes. The migration process used to represent this process is currently under development. The current migration method imposes a very low probability of chromosomes moving from one partition to another which resulted in a low probability of mixing tolled links from different partitions. It is also because the computational effort is shared between the tasks of finding the optimal toll location and toll levels. The number of SATURN runs used in this test is around 24,000 runs which is far less than 0.001% of the number of possible combinations of chromosome (approximately 5×10^{160} combinations). This implies that PGA-ALL may simply require more SATURN runs to approach a better result.

5. CONCLUSIONS AND FURTHER RESEARCH

We have demonstrated that the derivative based approach can solve the second-best tolling problem, but there are still some problems corresponding to the characteristics of MPEC. The GA-CHARGE approach was shown to be successful in solving the OPT2 problem giving slight improvements over the CORDON process.

The incremental LOCATE approach performed well in the case study of MINILEEDS, but in general suffers from the weakness whereby previously selected links cannot be de-selected when building a combination of toll points. The GA-LOCATE approach gives only a slight improvement in the case presented as many of the links selected by LOCATE are also in the GA solution. The problem OPT3 is the most difficult problem to solve. The structure of GA-LOCATEII should in theory be able to solve this problem, but errors in the location indices appear to be additive as the number of links considered is increased. The method based on PGA-ALL is then developed to bypass the use of location indices. However, the results of applying PGA-ALL with the MINILEEDS network suggested that it still needs further development.

Although, both GA-LOCATE and GA-LOCATEII produce better results than PGA-ALL, the method based on PGA is considered as a promising method. PGA-ALL is used to optimise the optimal toll location and toll levels at the same time which results in a massive number of combinations. At this stage, PGA is applied with its simplest structure. Several issues and techniques are still available for the improvement of the performance of PGA-ALL. In this paper, the candidate links are simply separated into 6 areas geographically. This appears to cause a problem in passing links from one partition to another. Further research is required in setting the partitions needed for parallelisation.

References

Abdulaal, M. and LeBlanc, L.J. (1979) “Continuous equilibrium network design models”. *Transportation Research* **13B**.

Allsop, R.E. (1974) “Some possibilities for suing traffic control to influence trip distribution and route choice,” in: *Proc. of the 6th International Symposium on Transportation and Traffic Theory*, ed: Buckley, D.J., Elsevier, New York.

Arnott, R.J., de Palma, A. and Lindsey R. (1990) “Departure time and route choice for the morning commuter”. *Transportation Research* **42B.**

Ben-Ayed, O. and Blair, C. (1990) “Computational difficulties of bilevel programming”. *Operations Research* **38**.

Ben-Ayed, O., Boyce, D.E. and Blair III, C.E (1988) “A general bilevel linear programming formulation of the network design problem”. *Transportation Research* **22B(4)**.

Berhendorff, P., Hearn, D.W., and Ramana, M.V. (1997) “Congestion Toll Pricing of Traffic Networks,” in Pardalos, P.M., Hearn, D.W., and Hager, W.W. (eds.) *Lecture Notes in Economics and Mathematical Systems Vol. 450*. Springer-Verlag.

Boyce D.E., LeBlanc, L.J., and Jansen, G.R.M. (1991) *Optimal toll stations in a road pricing system: an investigation into the feasibility of optimisation methods*. Report INRO (unpublished).

Cree, N.D., Maher, M.J., and Paechter, B. (1998) “The continuous equilibrium optimal network design problem: a genetic approach” in Bell, M.G.H. (eds.) *Transportation Networks: Recent Methodological Advances*. Pergamon, Amsterdam.

Davis, L. and Steenstrup M. “Genetic Algorithms and Simulates Annealing: An overview” in Davis, L. (eds.) *Genetic Algorithms and Simulated Annealing*. Morgan Kaufmann Publishers, Los Altos, California.

Friesz, T.L., Cho, H.J., Mehta, N.J., Tobin R.L., and Anadalingam, G. (1992) “A Simulated Annealing Approach to the Network Design Problem with Variational Inequality Constraints”. *Transportation Science* **26**.

Glazer, A. and Niskanen E. (1992) “Parking fees and congestion”. *Regional Science and Urban Economics* **22**.

Hearn, D.W. and Ramana M.V. (1998) “Solving Congestion Toll Pricing Models” in Marcotte, P. and Nguyen, S. (Eds.) *Equilibrium and Advances Transportation Modelling*. Kulwer Academic Publishers, London.

Hearn, D.W. and Yildirim, M.B. (2000) “A Toll Pricing Framework for Traffic Assignment Problems with Elastic Demand” [from http://www.ise.ufl.edu/hearn/ (4/02/02)]

Hyman, G. and Mayhew, L. (2002). “Optimizing the benefits of urban road user charging”. *Transport Policy*, in press.

Knight, F.H. (1924) “Some fallacies in the interpretation of social cost”. *Quaterly Journal of Economics* **38**.

LeBlanc, L.J. and Boyce D.E. (1986) “A bilevel programming algorithm for exact solution of the network design problem with user-optimal flows”. *Transportation Research* **20B(3)**.

Levy-Lambert, H. (1968) “Tarification des services a qualite variable: application aux peages de circulation”. *Econometrica* **36(3-4)**.

Liu, L.N. and McDonald, J.F. (1999) “Economic efficiency of second-best congestion pricing schemes in urban highway systems”. *Transportation Research* **33B**.

Marchand, M. (1968) “A note on optimal tolls in an imperfect environment”. *Econometrica* **36**.

Marcotte, P. (1983) “Network Optimization with continuous control parameters”. *Transportation Science* **17**.

Marcotte, P. (1986) “Network design problem with congestion effects: a case of bilevel programming”. *Mathematical Programming* **34**.

May A.D., Liu R., Shepherd, S.P., and Sumalee A (2002) “The impact of cordon design on the performance of road pricing schemes”. *Journal of Transport Policy*, forthcoming.

McDonald, J.F. (1995) “Urban highway congestion: an analysis of second-bet tolls”. *Transportation* **22**.

Meng, Q., Yang, H and Bell, M.G.H. (2001) “An equivalent continuous differentiable model and locally convergent algorithm for the continuous network design problem”. *Transportation Research* **35B**.

Michalewicz, Z. (1992) *Genetic Algorithms + Data Structures = Evolution Programs*. Springer-Verlag, New York.

Oscar Faber Consultancy. (2001) Road User Charging Study-West Midlands, Final Report prepared for Birmingham City Council. (unpublished)

Pigou, A.C. (1920) *Wealth and Welfare*. Macmillan, London.

Sharp, C.H. (1966) “Congestion and welfare, an examination of the case for congestion tax”. *Economic Journal* **76**.

Sheffi, Y. (1985) *Urban Transportation Networks: Equilibrium analysis with mathematical programming methods*. Prentice-Hall Inc., Englewood Cliffs, New Jersey.

Shepherd, S.P., May, A.D. and Milne, D.S. (2001a) “The design of optimal road pricing cordons”. *Proc. of the ninth world conference on transport research*, Seoul, Korea.

Shepherd, S.P., May A.D., Milne, D.S. and A. Sumalee (2001b) “Practical Algorithms for Finding the Optimal Road Pricing Location and Charges”. *Proc. of European Transport Conference, September 2001*, Cambridge, UK

Shepherd, S.P. and Sumalee A. (2002). “A genetic algorithms based approach to optimal toll level and location problems”. *Submitted to Network and Spatial Economics*.

Small, K.A. and Yan, J. (2001) “The Value of “Value Pricing” of Roads: Second-Best Pricing and Product Differentiation”. *Journal of Urban Economics* **49**.

Smith, M.J. (1979) “The existence, uniqueness and stability of traffic equilibrium”. *Transportation Research* **13B**.

Steenbrink, P.A. (1974) *Optimization of Transport Network*. John Wiley & Sons, London.

Sumalee A. (2001) “Analysing practical design criteria for charging cordons”. ITS Working paper No. 574, Institute for Transport Studies, University of Leeds, UK.

Suwansirikul, C., Friez, T.L., and Tobin, R.L. (1987) “Equilibrium decomposed optimisation: a heuristic for the continuous network equilibrium design problem”. *Transportation Science* **21**.

Tobin, R.L. and Friesz, T.L. (1983) “Sensitivity analysis for equilibrium network flow”. *Transportation Science* **22(4)**.

Van Vliet, D., 1982. SATURN – a modern assignment model. Traffic Engineering and Control **23(12)**.

Verhoef, E.T. (2000) Second-best congestion pricing in general networks: algorithms for finding second-best optimal toll levels and toll points. Discussion paper TI 2000-084/3, Tinbergen Institute, Amsterdam-Rotterdam.

Vickrey, W.S. (1969) “Congestion theory and transport investment”. *American Economics Review* **59**.

Walters, A.A. (1961) “The theory and measurement of private and social cost of highway congestion”. *Econometrica* **29(4)**.

Wardrop, J. (1952) “Some theoretical aspects of road traffic research”. *Proc. of the Institute of Civil Engineers* **1(2)**.

Wilson, J.D. (1983) “Optimal road capacity in the presence of unpriced congestion”. *Journal of Urban Economics* **13**.

Whitley, D. (1989) The GENITOR algorithm and selection pressure : Why rank based allocation of reproductive trial is best. In Schaffer, J. (Eds) Proceeding of the 3rd International Conference on Genetic Algorithms, Morgan Kaufmann, Publishers, CA, USA.

Yang, H., Zhang, X. and Huang, H.J. (2002) “Determinations of optimal toll levels and locations of alternative congestion pricing schemes”. Proc. of the 15th International Symposium on Transportation and Traffic Theory, Adelaide, Australia.

Yang, H. (1997) “Sensitivity analysis for the elastic-demand network equilibrium problem with applications”. *Transportation Research* **31B**.

Yang, H. and Bell, M.G.H. (1997) “Traffic restraint, road pricing, and network equilibrium”. *Transportation Research* **31B**.

Appendix A: Notation

N	The set of nodes in the network
I	the set of OD-pairs, denoted i=1,…,I
T_i	the continuous number of users (or OD-flow) for OD pair i, with $T_i \geq 0$
$D_i(T_i)$	the inverse demand function for trips for OD-pair i, with $D'_i \leq 0$
J	the set of directed links in the network, denoted j=1,..,J
V_j	the continuous number of users (or link flow) on link j, with $V_j \geq 0$
$C_j(V_j)$	the average cost function for the use of link j, with $c_j' \geq 0$
Cp	the travel costs on path p
$\prod$	the set of non-cyclical paths in the network, denoted p=1,…,P
F_p	the continuous number of users (or path-Flow) for path p, with $F_p \geq 0$
$\prod_I$	the set of non-cyclical paths for OD-pair i, denoted p_i=1,…,P_i
δ_{jp}	A dummy that takes on the value of 1 if link j belongs to path p, and a value of 0 otherwise
ε_j	A dummy that takes on the value of 1 if a toll can be charged on link j, and a value of 0 otherwise
f_j	the level of the toll on link j if ε_j=1
i or k	index for OD pairs
j or m	index for links
p or q	index for paths
λ_p	Lagrange multiplier associated with path p
Δ_{ip}	A dummy equal to 1 if $p \varepsilon \prod_i$ and