A BAYESIAN APPROACH TO MODELLING UNCERTAINTY IN TRANSPORT INFRASTRUCTURE PROJECT FORECASTS

Kevin Cheung
Imperial College Business School
Imperial College London

John W. Polak
Centre for Transport Studies
Imperial College London

Abstract

The increased participation of the private sector in the delivery of transport infrastructure projects has increased the emphasis on understanding the accuracy and uncertainty of traffic demand forecasts. Transport models which provide these forecasts rely on simplified assumptions usually involving a combination of physical, socioeconomic, environmental and individual factors for a modelled base and future time period. Uncertainty in the value of input parameters and their conditional relationships results in uncertainty in the outturn forecasts. The accuracy of model predictions is normally tested through a number of quantitative and statistical methods. This paper presents a summary of the approaches used to model uncertainty in practice including scenario testing, sensitivity testing and statistical risk analysis using Monte-Carlo methods. However, other techniques are now available, and may offer superior insight into the structure of the underlying problem. In this paper, Bayesian belief networks, together with Monte Carlo Markov Chain techniques, are applied as an alternative method for modelling uncertainty in transport modelling. We illustrate the technique on a simplified toll road case study, based on a motorway in São Paulo, Brazil, in which we compute equilibrium solutions for traffic flow, travel time and cost for fixed demand and elastic demand problem formulations. The paper concludes on a comparison between the Bayesian belief network and a more conventional sensitivity analysis and discusses the relative merits of each approach.

Keywords: Bayesian methods, Bayesian networks, Toll Roads, Uncertainty, Traffic demand, Monte-Carlo analysis

1 Introduction

The traffic forecasts produced by transport models are subject to a number of sources of uncertainty including errors in the measurement of input data, errors in the estimated value of model parameters and errors in the specification of the underlying models themselves. Ideally, analysts would wish to understand the separate and collective impact of these errors on the uncertainty of model forecasts, so as to be able to attach credible confidence intervals to model forecasts and optimise the allocation of study resources. However, in large model systems, the interaction between each of these sources of error can be very complex, making the analysis of propagation of uncertainty through the modelling process extremely challenging. Nevertheless, the increased participation in recent years of the private sector in the delivery of transport infrastructure projects has raised the requirement for accurate traffic demand forecasts and led to renewed interest in the analysis of model uncertainty.

The classical approaches to addressing these issues in the transport literature include scenario analysis, sensitivity testing and statistical risk analysis using Monte-Carlo methods. However, all these approaches have significant limitations. In this paper we propose a new approach which is based on Bayesian statistical principles which we believe offers superior insight into the structure of the underlying problem. Our approach involves expressing the transport demand model as a Bayesian belief network (BBN). BBNs are powerful tools for Bayesian statistical inference, which enable the representation of causal dependencies between sets of random variables and the computation of the joint posterior distribution of these random variables, conditional on prior distributions of each variable and data. The computation of the posterior joint distribution is performed using Markov Chain Monte Carlo (MCMC) methods, which involve approximating the desired joint posterior distribution (which is generally analytically intractable) by a series of simpler distributions, formed from the product of tractable conditional distributions. This technique permits the flexible representation of a range of different sources of uncertainty and the consistent propagation of this uncertainty through the model system.

The paper is divided into a number of sections. Following the introduction, the second section provides a brief overview of the existing literature on the modelling of uncertainty in transport modelling, highlighting the strengths and weaknesses of the different approaches. The third section briefly introduces the BBN methodology and discusses some of the specific issues associated with its application to typical transport modelling contexts. The fourth section presents an application of the BBN approach to a toll road case study in São Paulo, Brazil. In this application we explicitly formulate a simple equilibrium transport model as a BBN and use this formulation to explore the propagation uncertainty in traffic data and model parameters into revenue forecasts. The fifth section summarises a number of test results from the model including a comparison between the BBN and a more conventional sensitivity analysis and discusses the relative merits of each approach. The paper concludes with a summary of the main findings and directions for future research.

2 Transport Model Forecast Uncertainty

2.1 Forecasting Risk

Transport infrastructure projects generally require a large capital investment from either the public or private sector. As with any investment there is an element of risk, and a transport infrastructure enterprise is no exception. Transport infrastructure project finance has developed rapidly over the last 20 years and risk evaluation – commercial, macro-economic and political – is at its heart (Yescombe, 2002). A key input into the business and financial case for these projects are project cash flows which, for most projects, are related to forecast traffic demand. Inaccurate traffic forecasts (Flyvbjerg et al., 2006; Standard & Poor's, 2002, Bain et al., 2005) can lead to project failures and bankruptcies (World Bank, 2006).

2.2 Transport Project Forecasts

Traffic forecasts are produced by transport models. These models are created from data sources, such as surveys, and attempt to capture 'typical traffic conditions' of a particular project in a base model. Future time-period models are then created based on forecast changes in explanatory variables. These project forecasts are normally provided as point estimates with no direct reference to their level of variability, or uncertainty.

Uncertainty in transport model forecasts arises broadly from two error sources: the model inputs and the models themselves (De Jong et al., 2005). Where input uncertainty can include the future change in socio-economic variables or other exogenous factors; and model uncertainty can include specification error or error due to using parameter estimates instead of true values. A further distinction can be applied between those factors that affect the base traffic model and those that affect the future model forecasts. Table 2.1 shows the common types of input and model specification uncertainties.

Table 2.1: Input and Model Uncertainties by Project Modelled Years

Base Year Factor	Future Year Factors
Quality of base data/traffic flows	GDP
Matrix development/estimation	GDP/car ownership elasticity
Model specification	Growth in value of time
Estimates of journey time savings	Toll tariff
Value of time estimates	Growth in toll tariff
Assignment/route choice techniques	Effect of other road schemes
Model calibration parameters	Induced traffic
	Traffic ramp-up

Source: (Brett et al., 1999; Boyce, 1999; Boyce et al., 2003)

The uncertainty embedded in individual input factors and in model structure is propagated into the uncertainty in overall traffic forecasts. Error propagation depends on the structure of the underlying transport model. In one of the few studies to systematically investigate this question, Zhao and Kockelman (2001) quantified uncertainty propagation through a conventional four-stage transport demand model process (trip generation, trip distribution, mode choice and route assignment). Through comparison of model outputs from input variation, the research concluded that results from transport demand models may be highly uncertain. Uncertainty was found to compound over the four modelling stages of transport travel demand.

2.3 Approaches to Modelling Uncertainty in Transport Model Forecasts

There are a number of general approaches to assess the degree of forecasting risk in any type of project or investment analysis. These include scenario testing, sensitivity testing and risk analysis (Ross et al., 2008).

The most basic form of analysis used to assess uncertainty in traffic forecasts is scenario testing. This is where a number of alternative scenarios are developed, based on the simple variation of key model assumptions around a “central case” (or “base case”) forecast. Sensitivity analysis is a variation on scenario testing which is a series of model runs for a range of input parameters and used to pinpoint the variables where forecasting risk is most severe.

More advanced simulation analysis attaches confidence intervals to the forecasts, by assuming statistical distributions for the model input parameters. This method is referred to as risk or Monte Carlo analysis and is based on statistical sampling theory. Monte Carlo analysis models the consequences of uncertainty in inputs and correlations between these inputs. It involves replacing point values with probability distributions of possible values for key inputs. Typically, the choice of probabilistic inputs will be based on prior sensitivity testing. Sampling is then repeated randomly a large number of times. The results consist of a set of probability distributions showing how uncertainties in key inputs might impact on key outcomes. Monte Carlo sampling is described in traffic and revenue forecasting applications in De Jong *et al.* (2005), Brett and Snelson (1999) and Boyce and Bright (2003).

There are shortcomings to the scenario, sensitivity and conventional risk analysis approaches. No developed methodology or model incorporates these uncertainties through the traffic model system. Instead, conventional risk analysis relies upon obtaining a relationship between model inputs and outputs based on sensitivity testing and combining these relationships. Other techniques are available that may offer superior insight into the structure of the underlying problem. This paper demonstrates an alternative approach using BBN, together with MCMC techniques, for modelling uncertainty in transport modelling and forecasting.

3 Bayesian Analysis for Modelling Uncertainty

Bayesian analysis is an important branch of statistics based on Bayes' theorem (Bayes, 1793; Raiffa, 1968). This theorem relates the prior and posterior probabilities for stochastic events. The theorem states how prior probabilities, combined with new information from a test or sample, can be used to update or revise beliefs to yield posterior probabilities. Given a prior distribution for the unknown parameter θ – representing previous knowledge or belief – and the observed sample data x, a posterior distribution of θ, is calculated as:

$$p(\theta \mid x) = \frac{p(x \mid \theta)\, p(\theta)}{p(x)}$$

where: θ is the unknown parameter

p(θ) is the prior distribution of θ

x is a sample drawn from underlying distribution

p(*x*|θ) is the sampling density of *x*

p(θ|*x*) is the posterior distribution of *x*

The sampling density $p(x|\theta)$ is known as the likelihood function with x fixed as θ varies. Thus, the posterior distribution for θ takes into account both prior distribution for θ and the prior probability of observed data x.

A BBN is a probabilistic graphical model and is a way of formalising probabilistic dependencies between a set of variables. It is a powerful tool for modelling probabilistic inference. The approach calculates posterior distributions based upon a representation of causal dependencies to random variables of interest (Jensen, 1996;Montironi et al., 1996; Gilks et al., 1996). BBNs have been used as a means of modelling causal uncertainty in a diverse range of domains in industry, computing, services and natural and social sciences (Pourret et al., 2008; Scollnik, 2001; Berger, 2000)

BBN analysis emerged as a result of mathematical research carried out in the 1980s (Pourret et al., 2008), but was computationally demanding. Advances in the area of approximation methods in the early 1990s (Scollnik, 2001), including the emergence of computer intensive MCMC sampling methods, allowed Bayesian analysis to be applied to a wide range of problems.

The MCMC algorithms provide a set of statistical tools that enable the estimation of potentially complex statistical models based on probabilistic inference (Berger, 2000). MCMC methods exploit the relationships of conditional independence encoded in a graphical model such as a BBN to enable samples to be drawn from complex underlying distributions by means of sampling sequentially from simpler component distributions. MCMC methods were first introduced by Metropolis et al. (1953) and subsequently developed by Hastings (1970). A variety of different sampling strategies have been developed, of which the most commonly used are the Metropolis-Hastings algorithm and the Gibbs sampler (Brooks, 1998; Geman et al., 1984)

In the field of transport studies, Verhoeven et al. (2004) used a BBN to mode choice and Lindveld et al. (2006) used BBNs to perform data fusion. However, to date there appear to have been no applications of BBN techniques to study the propagation of uncertainty in transport models.

4. Deterministic Model Example Application to Toll Roads

4.1 Toll Road Case Study: São Paulo Rodoanel Oeste

A simple deterministic traffic model was developed to characterise and model uncertainty for a typical toll road context. The simplified case study is based on the Rodoanel Oeste[1] in São Paulo, Brazil. Figure 4.1 shows the Rodonael Oeste toll road and its free congested competing route.

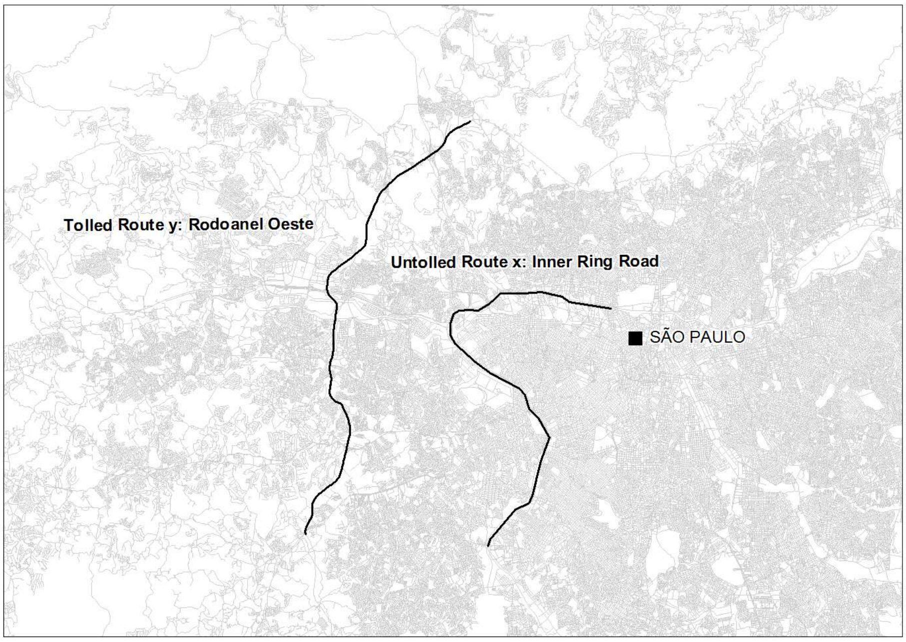

Figure 4.1: Simplified Toll Road Case Study: São Paolo Rodoanel

In conventional transport models, a network equilibrium solution between transport supply and demand is found through the redistribution of trips in an iterative distribution and assignment algorithm. In order to allow the application of a BBN, a model was developed that could compute a deterministic equilibrium solution algebraically.

[1] The Rodoanel Oeste is a 32km motorway which is operated by Concessionária do Rodoanel Oeste under terms of a 30-year PPP contract with the State of São Paulo. The toll road operation started in December 2008 after a competitive bid process.

4.2. Notation

v_{curr}	– actual speed	[km/h]
v_0	– free flow speed	[km/h]
L_x	– length of route X	[km]
L_y	– length of route Y	[km]
t_{curr}	– actual travel time on a route	[h]
t_0	– free flow travel time on a route	[h]
Q	– total travel demand	[veh]
α	– demand constant	[-]
β	– demand constant	[-]
VOT	– value of time for travellers	[$/h]
x	– flow on route X	[veh]
y	– flow on route Y	[veh]
x_{max}	– capacity of route X	[veh/h]
y_{max}	– capacity of route Y	[h/veh]
φ_x	– capacity reciprocal of route X	[h/veh]
φ_y	– capacity reciprocal of route Y	[veh/h]
$TOLL_x$	– toll per km on route 1	[$/km]
$TOLL_y$	– toll per km on route 2	[$/km]
c_x	– cost of travelling on route X	[$]
c_y	– cost of travelling on route Y	[$]
C	– travel cost at equilibrium	[$]

4.3. Model Specification

4.3.1 Supply

The case study considered here represents the transport supply side of the model by a road network that consists of two links: untolled (*x*) and tolled (*y*) routes respectively. The generalised link costs are, in part, a function of a number of physical attributes and tolls. All time and monetary costs were converted into generalised costs through the application of an assumed value of travel for road users. The cost of travelling on a road is defined as the cost of time required to travel the route and the cost of tolls, hence:

- Cost of time = Actual travel time × Value of time
- Monetary cost = Toll per 1 km × Length of route

The cost of travelling along the length of Route *X* and *Y*:

Route X: $$c_x = t_{curr}^x VOT + TOLL_x L_x \quad (1)$$

Route Y: $$c_y = t_{curr}^y VOT + TOLL_y L_y \quad (2)$$

Congestion is considered through a volume-delay function (VDF) that determines generalised cost. A VDF based on a simple deterministic queuing model (Newell,

1971) was utilised due to its comparatively simple form. The total generalised cost of travelling along routes *x* and *y* were thus defined as follows:

Route X: $$c_x = \frac{t_0^x VOT}{1-\varphi_x x} + TOLL_x L_x \quad (3)$$

Route Y: $$c_y = \frac{t_0^y VOT}{1-\varphi_y y} + TOLL_y L_y \quad (4)$$

4.3.2 Demand

Transport demand in this study was modelled for two scenarios:

- Scenario 1: Fixed Traffic Demand – A fixed number of trips to be distributed between the two road choices

$$Q = constant \quad (5)$$

- Scenario 2: Elastic Traffic Demand – A more complicated but realistic case is the assumption of elastic demand. Assuming that the highway system is governed by the principles of economic theory, transport supply can be considered as a standard economic good and, therefore, as network costs change so will traffic demand

$$Q = \alpha - \beta \cdot C \quad (6)$$

4.3.3 Network Equilibrium

Scenario 1: Equilibrium with Fixed Demand

Equilibrium conditions, defined by Wardrop (1952), require that for a given origin-destination pair with two possible routes the cost of travelling along both routes is equal.

For a fixed travel demand:

$$Q = x + y \quad (7)$$

this corresponds to:

$$C_x = C_y \quad (8)$$

Substituting (3), (4) and solving it simultaneously with (8):

$$\frac{t_0^x VOT}{1-\varphi_x x} + TOLL_x L_x = \frac{t_0^y VOT}{1-\varphi_y (Q-x)} + TOLL_y L_y \quad (9)$$

Equation (9) reduces to a quadratic equation of the form $Ax^2 + Bx + C = 0$. Solving the quadratic equation, with respect to *x*, (traffic flow on untolled route) allows the calculation of *y* (traffic on tolled route) using equation (7). Although in most cases the quadratic equation has two solutions, non-negativity of traffic flow requires *x* > 0, which for most input values is unique.

Scenario 2: Equilibrium with Elastic Demand

When the demand *Q* is considered to be a function of the unique (equilibrium) origin-destination travel cost *C* as in equation (6), the Wardropian equilibrium solution between transport supply and demand is the simultaneous solution of equations (6), (7) and (8).

Using appropriate choice of values of model inputs, the equilibrium solutions to the elastic demand problem can be found through an equivalent optimisation formulation (Evans, 1976). Suppose that Q^{-1} is an inverse demand function (the function that gives equilibrium travel cost as a function of demand). From (6) we compute:

$$C = \frac{\alpha - Q}{\beta} \tag{10}$$

and obtain:

$$Q^{-1}[Q(C)] = C(Q) = \frac{\alpha - Q}{\beta} \tag{11}$$

We define the objective function *Z* as a sum of definite cost integrals. In the case of the elastic demand model, *Z* is minimised subject to demand conservation constraints and with respect to *x* and *y*:

$$\mathit{Min}\ \ Z = \int_0^x c(x)dx + \int_0^y c(y)dy - \int_0^Q C(Q)dQ \tag{12}$$

Hence, in the case of the simple elastic demand model the objective function *Z* is:

$$Z = \int_0^x \left[\frac{t_0^x VOT}{1 - \varphi_x x} + Toll_x L_x \right] dx + \int_0^y \left[\frac{t_0^y VOT}{1 - \varphi_y y} + Toll_y L_y \right] dy - \int_0^Q \left[\frac{\alpha - Q}{\beta} \right] dQ \tag{13}$$

Solving the integrals and calculating their value for the upper non-zero limits gives:

$$Z = \frac{t_0^x VOT \ln(1 - \varphi_x x)}{-\varphi_x} + Toll_x L_x x + \frac{t_0^y VOT \ln(1 - \varphi_y y)}{-\varphi_y} + Toll_y L_y y - \frac{\alpha Q - \frac{1}{2} Q^2}{\beta} \tag{14}$$

Substituting equation (7) into the last components of equation (14) we obtain:

$$Z = \frac{t_0^x VOT \cdot \ln(1-\varphi_x x)}{-\varphi_x} + Toll_x L_x x + \frac{t_0^y VOT \cdot \ln(1-\varphi_y y)}{-\varphi_y} + \\ Toll_y L_y y + \frac{1}{2\beta}(x^2 + 2xy + y^2) - \frac{\alpha}{\beta}(x+y) \tag{15}$$

Function *Z* achieves minimum when both partial derivatives are equal to zero:

$$\frac{\partial Z}{\partial x} = \frac{t_0^x VOT}{1-\varphi_x x} + Toll_x L_x - \frac{x+y}{\beta} - \frac{\alpha}{\beta} = 0 \tag{16}$$

$$\frac{\partial Z}{\partial y} = \frac{t_0^y VOT}{1-\varphi_y y} + Toll_y L_y - \frac{x+y}{\beta} - \frac{\alpha}{\beta} = 0 \tag{17}$$

and when second order derivatives with respect to *x* and *y* are non-negative, and the following condition is true:

$$\frac{\partial^2 Z}{\partial x^2} \cdot \frac{\partial^2 Z}{\partial y^2} = \left(\frac{\partial^2 Z}{\partial x \partial y} \right)^2 > 0 \tag{18}$$

With the second order derivatives of function *Z* as follows:

$$\frac{\partial^2 Z}{\partial x^2} = \frac{\varphi_x t_0^x VOT}{(1-\varphi_x x)^2} - \frac{1}{\beta} \tag{19}$$

$$\frac{\partial^2 Z}{\partial y^2} = \frac{\varphi_y t_0^y VOT}{(1-\varphi_y y)^2} - \frac{1}{\beta} \tag{20}$$

$$\frac{\partial^2 Z}{\partial x \partial y} = -\frac{1}{\beta} \tag{21}$$

The conditions for the existence of minimum correspond to:

$$\frac{(1-\varphi_x x)^2}{\varphi_x t_0^x VOT} < \beta \tag{22}$$

$$\frac{(1-\varphi_x x)^2}{\varphi_x t_0^x VOT} < \beta \quad (23)$$

$$\left(\frac{\phi_x t_0^x VOT}{(1-\varphi_x x)^2} - \frac{1}{\beta}\right)\left(\frac{\varphi_y t_0^y VOT}{(1-\varphi_y y)^2} - \frac{1}{\beta}\right) - \left(\frac{1}{\beta}\right)^2 > 0 \quad (24)$$

The argmin of the *Z* function can be derived by setting $\frac{\partial Z}{\partial x} = \frac{\partial Z}{\partial y} = 0$, from which *y* can be yielded as a function of *x*:

$$y = \frac{1}{\varphi_y}\left(1 - \frac{t_y VOT}{\left(\frac{t_x VOT}{1-\varphi_x x}\right) + Toll_x L_x - Toll_y L_y}\right) \quad (25)$$

Substituting equation (25) back into equation (17) we obtain an equation that allows the calculation of *x*:

$$\frac{t_x VOT}{1-\varphi_x x} + Toll_x L_x + \frac{\dfrac{1 - \dfrac{t_y VOT}{\dfrac{t_x VOT}{1-\varphi_x x} + Toll_x L_x - Toll_y L_y}}{\varphi_y} + x}{\beta} - \frac{\alpha}{\beta} = 0 \quad (26)$$

To solve for *x*, a number of simplifying substitutions are made to equation (26) before solving the cubic equation. Given that the solution must be non-negative a unique x can be found under mild conditions, with corresponding *y* derived using equation (25). Full details of the derivation can be found in Cheung (2008)

4.4 Deterministic Model Results

The models were first programmed deterministically with realistic values of input variables ensuring existence of an initial feasible solution. These input values made reference to ARTESP (2007) and DERSA (2008) with model outturn results comparing well with observed data. Table 4.1 shows the adopted values of inputs and outputs for the fixed demand model.

Table 4.1: Fixed Demand Model Inputs and Outputs

	Variable	Untolled Route X: Inner Ring Road	Tolled Route Y: Rodoanel Oeste
Input variable	VOT (Real$/hr)	20	
	Total Demand (veh)	10000	
	Free-flow Speed (km/hr)	80	120
	Length (km)	25	32
	Number of lanes per direction	3	2
	Capacity (veh/hr)	11400	7600
	Toll (Real$/km)	0	0.2
Output variable	Demand (veh/hr)	6822	3178
	Time (hr)	0.8	0.5
	Speed (km/hr)	32	70
	Equilibrium cost (Real$)	15.6	15.6

4.5 BBN Model: Example Application to Toll Roads

The software program WinBUGS (Lunn et al., 2000) was used to perform the Bayesian Analysis using MCMC sampling methods. The program allowed BBN models to be graphically specified in the form of directed networks. The acyclic nature of the inference process dictated the type of simplified model that could be developed for the analysis.

Having specified the deterministic traffic models in WinBUGS, values for model input variables were defined together with any assumed distribution. WinBUGS uses MCMC methods to calculate sample values on all the unknown values from their conditional (posterior) distribution given those stochastic nodes that have been specified. The MCMC method randomly samples from the prior distributions of all stochastic nodes for each simulation.

Figures 4.2 and 4.3 show the WinBUGS fixed and elastic demand models respectively.

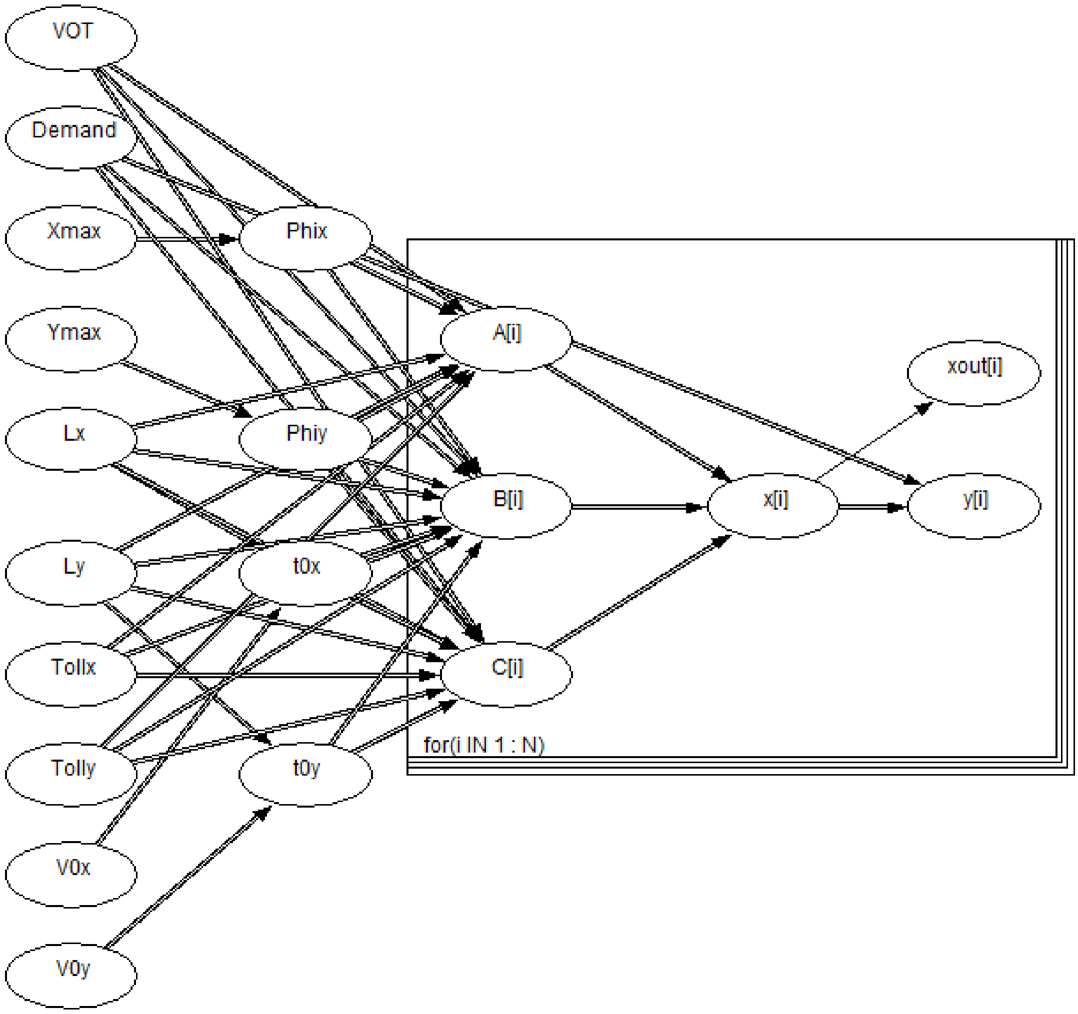

Figure 4.2: WinBUGS Model for Fixed Demand

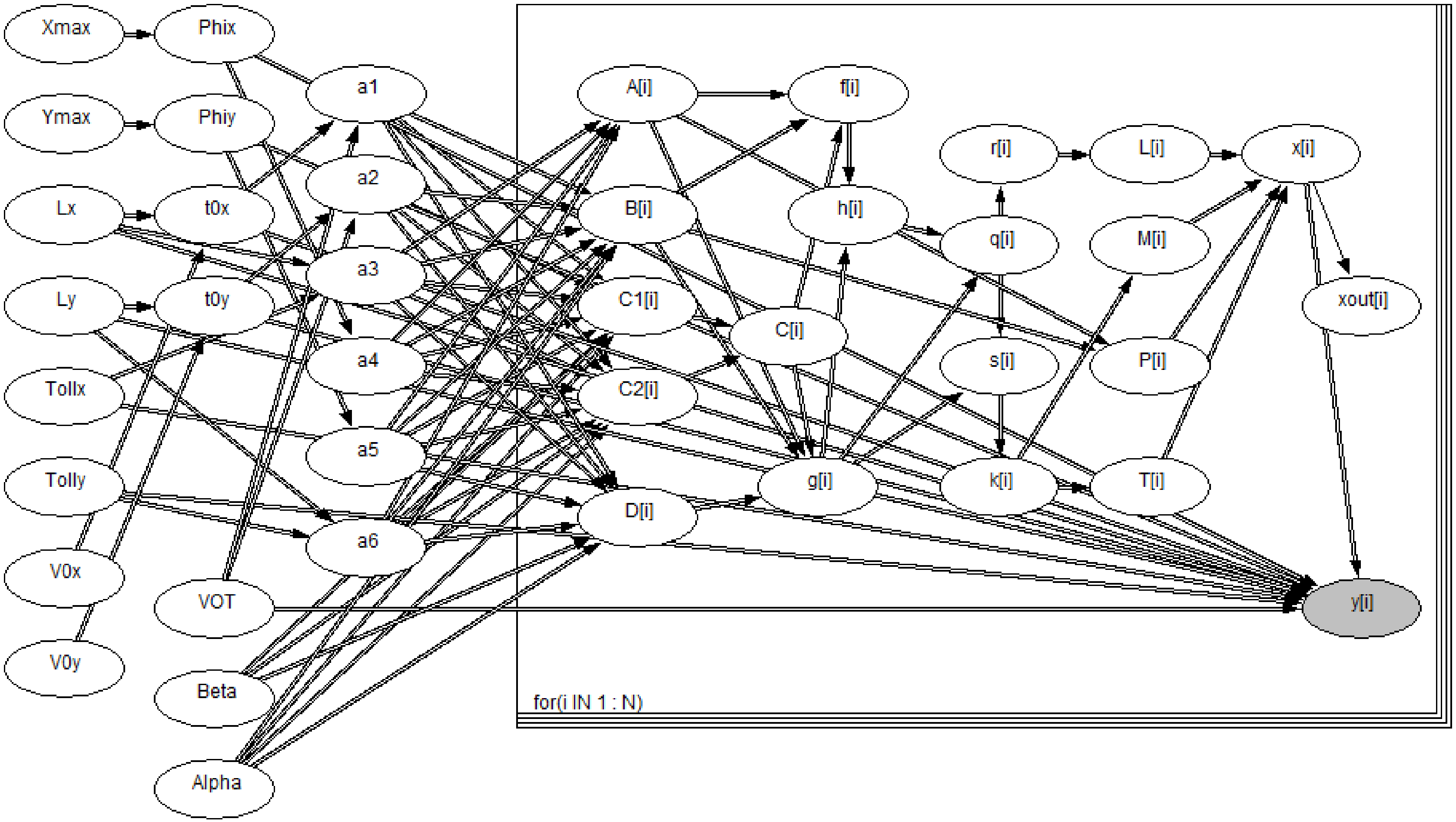

Figure 4.3: WinBUGS Model for Elastic Demand

5 Model Tests

The models were used to investigate the following:

1. VOT variable uncertainty propagation;
2. Demand parameter uncertainty propagation;
3. Monte Carlo Analysis Comparison.

The results of the model tests summarise posterior distributions in terms of Bayesian probability percentiles, mean, standard deviation and the percentage error defined as follows:

$$\text{Percentage Error} = \left(\frac{\text{Standard Deviation}}{\text{Mean}}\right)100$$

5.1 VOT Variable Uncertainty Propagation

VOT is one of the most important input variables in road transport modelling. To determine how the variation in this value affected the output traffic levels on both untolled and tolled routes, VOT was defined as a stochastic variable with a generalised gamma distribution. The scale of this distribution was set with reference to the existing literature (Armstrong et al., 2001; Gaudry et al., 1989).

When VOT is stochastic, with all other variables deterministic, the overall shape of VOT distribution is propagated through to the posterior distributions of traffic on routes x and y. By changing the level of uncertainty in the VOT distribution as shown on Figure 5.1, the sensitivity of posterior traffic estimates to VOT uncertainty are shown on Figure 5.2. The results show an asymmetry in the propagation of uncertainty to the posterior traffic estimates. This is related to the asymmetry in the gamma distribution of VOT.

This result for the fixed demand model is replicated in the result for the elastic demand model, hence providing a check on the algebraic equilibrium solutions. It can be expected that by varying β, the demand slope changes and hence so does the equilibrium solution of traffic distribution.

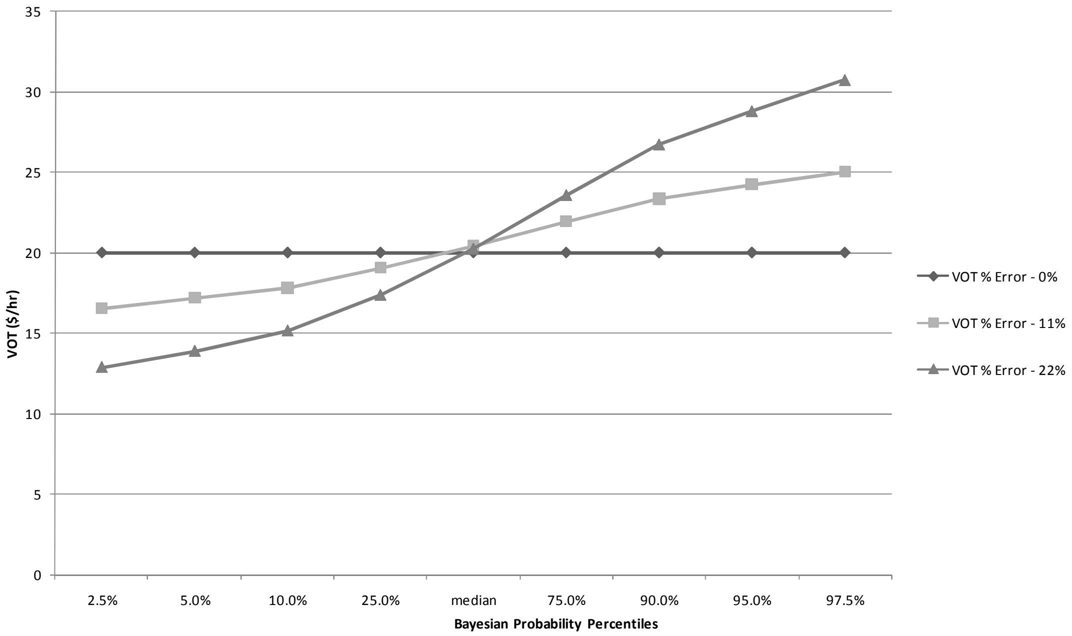

Figure 5.1: Variation in VOT Percentage Error

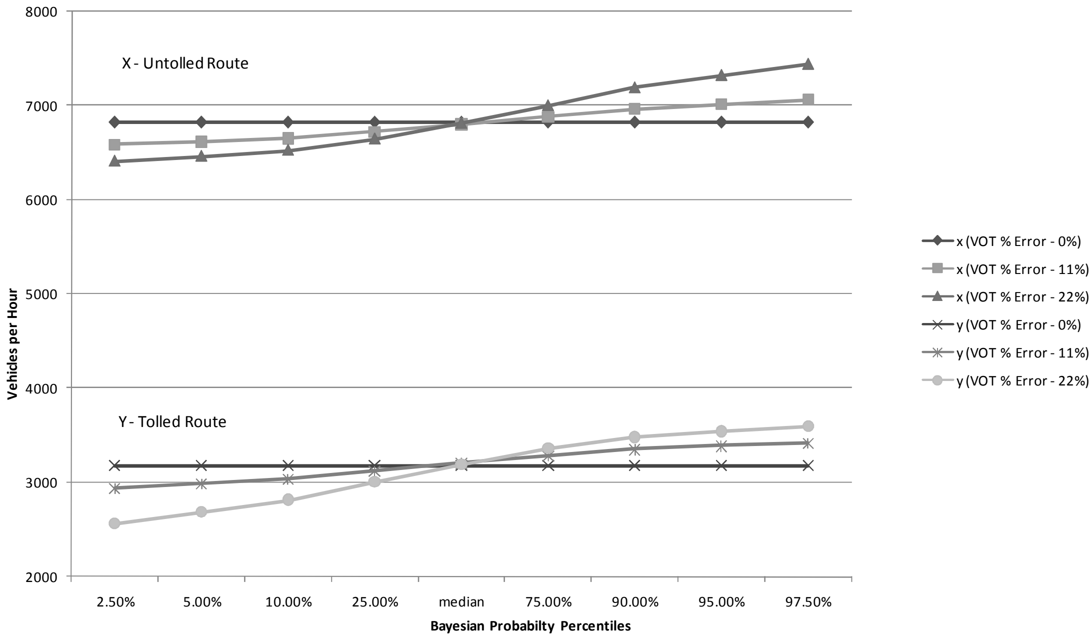

Figure 5.2: Variation of Outturn Traffic with Variation in VOT Percentage Error

5.2 Demand Parameter Uncertainty Propagation

A test was undertaken using the elastic demand model to examine uncertainty in initial level of demand α and parameter β whilst keeping all other variables fixed. The α and β demand parameters utilised conservative uncertainty assumptions. After 10,000 MCMC simulations, the resulting distributions for tolled and untolled routes are shown in Figure 5.3, similar in shape but different in spread to the demand parameter distributions.

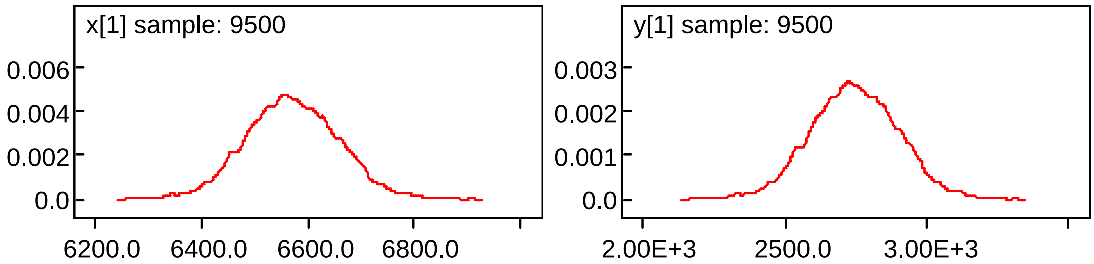

Figure 5.3: Bayesian Posterior Distribution for Untolled and Tolled Routes with Variation in α and β

The percentage error around the tolled route is significantly greater than the untolled route – a product of the model specification. The inclusion of β, at the assumed mean value and with uncertainty, does not significantly impact the resultant equilibrium traffic flows or distributions.

5.3 Monte Carlo Analysis Comparison

A conventional risk analysis using Monte Carlo simulation software was applied to the deterministic traffic model. The result of this analysis was compared against the equivalent model result using the BBN with MCMC approach developed in WinBUGS. The elastic demand model was programmed analytically in Microsoft Excel and Monte Carlo sampling applied using the Excel add-in software @Risk (Palisade, 2002). The deterministic model was set-up to test a stochastic VOT parameter on equilibrium traffic. The VOT parameter was given a probability distribution near identical to that assumed in the equivalent WinBUGS model and Monte Carlo sampling was undertaken for 10,000 simulations. The results for the VOT uncertainty test are comparable between both WinBUGS and conventional Monte Carlo analysis. This is because both methods are applied to a deterministic case that forms an analytical relationship between inputs and outputs.

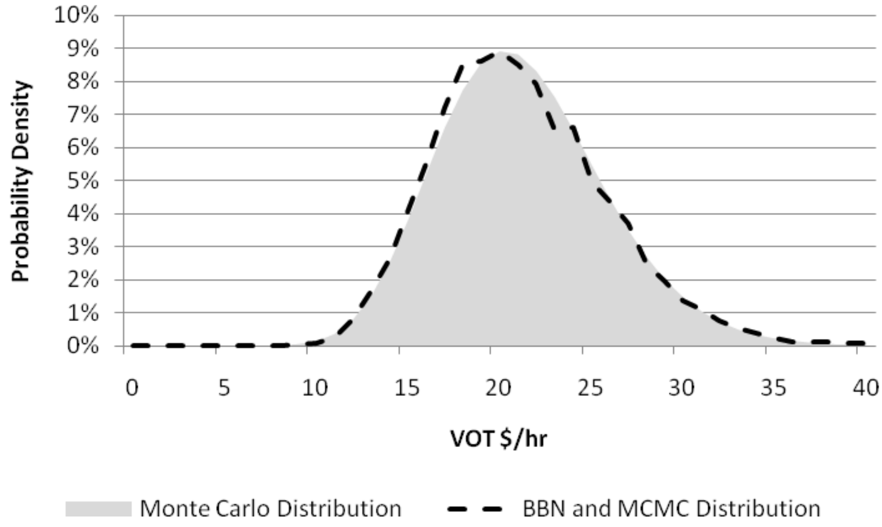

Figure 5.4: VOT Distribution Comparison between BBN with MCMC and Conventional Monte Carlo Analysis

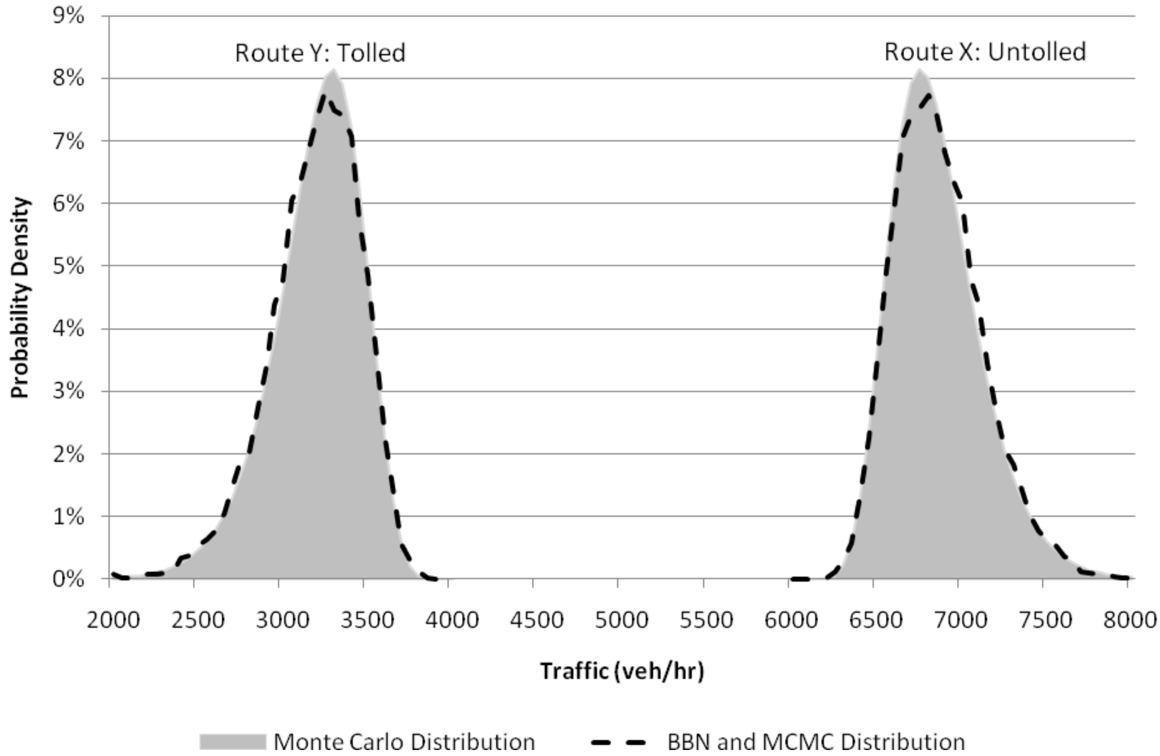

Figure 5.5: Traffic Distribution Comparison between BBN with MCMC and Conventional Monte Carlo Analysis

Figure 5.4 compares the distribution of the BBN and conventional Monte Carlo analysis obtained for VOT variable. The distribution comparison graphs for outturn untolled and tolled route traffic, as shown in Figure 5.5, are similar in overall shape to the VOT distribution but the distribution skew is noticeably more pronounced; this is related to the gamma distribution of VOT. The distribution plots corroborate the results and show how uncertainty propagation from VOT uncertainty to traffic uncertainty is related to the analytical model structure and level of input uncertainty. This test serves to demonstrate that both methodologies when applied to the same analytical problem formulation produced similar results. Whilst the sampling methodology is equivalent between methodologies, the differences lie in the characterisation of the problem, whether a BBN or more simply a relationship between inputs and outputs. Where the relationship between inputs and outputs cannot be analytically formulated, the conventional Monte Carlo approach would rely upon sensitivity testing.

6 Conclusions

In the test examples, the distribution of VOT input uncertainty was transferred to the outputs with traffic uncertainty at a lower order of magnitude. By varying the demand curve of the elastic model the more important parameter was found to be the α value – the initial demand level – rather than the β value – the slope of the demand curve. In the cumulative uncertainty test applied to the elastic demand case, where all significant input variables were defined stochastically, the resultant traffic uncertainty was more pronounced especially in the tolled road traffic.

In comparing the BBN with MCMC approach with conventional Monte Carlo analysis, the results of both approaches are similar when applied to the same analytical traffic models. This result confirms the validity of the underlying traffic models and also demonstrates that both methods can be used to investigate uncertainty propagation in traffic models.

The main advantage of the BBN with MCMC is its flexibility. The network is suitable to investigate and characterise the impact of stochastic variables and how they propagate through the model. The definition of the BBN also helps to formulate the model by forcing the specification of model variable dependencies. Another advantage of the Bayesian approach is the possibility of combining prior beliefs with empirical data to make inferences. The developed models did not utilise additional datasets, and therefore the full functionality of the Bayesian approach, but instead relied upon the prior distribution assumptions of the input variables.

There are limitations to the BBN with MCMC approach as demonstrated in this paper:

- The initial complexity of network development
- The downsides of network flexibility
- The current limitation to the analysis of simplified problems

The specification of the network can be complex both in terms of developing the conditional relationships and in computer programming. This limits the practical application of the technique to larger problem formulations.

The flexibility of the BBN to analyse variation and uncertainty propagation within the model scope is one of its advantages. However, this can also lead to model specification issues where the question formulation becomes critical in the targeted investigation of uncertainty.

The use of BBNs with MCMC is demonstrated for a theoretical case study. An obvious recommendation is to apply the approach to a more realistic case study, to include forecast year growth factors, or within a four-stage modelling approach – although current technology may render this application commercially impractical.

Future areas of research can develop or extend the existing networks and include investigating:

- model uncertainties from use of different forms of VDFs in equilibrium models;
- future year traffic growth variables on forecast traffic;
- stochastic assignment traffic models; and
- network model applications.

BBN models offer an alternative approach to incorporating uncertainty in transport models. The primary objective of this paper was to demonstrate the approach when applied to a simple toll road case study, developed from traffic modelling principles. Although practical applications are currently limited, modelling uncertainty in the traffic model process provides essential information to decision-makers and this approach offers an exciting area of research.

References

Armstrong, P., Garrido, R., and Ortúzar. J. d. D. (2001) Confidence intervals to bound the value of time, Transportation Research Part E 37 (2001) 143-161

ARTESP (2007) Rodoanel Maria Covas; Estudo de Viabilibade Financeira da Operaçao Privada do Trencho Oeste do Rodoanel e Cálculo do Valor da Outorga, Relatório Téchnico.

Bain, R. and Polakovic, L. (2005) Traffic Forecasting Risk Study Update 2005 Through Ramp Up and Beyond, Standard & Poor's, European Transport Conference Proceedings

Bayes, T. (1763) An Essay towards solving a Problem in the Doctrine of Chances. By the late Rev. Mr. Bayes

Berger, J. O. (2000) Bayesian Analysis: A Look at Today and Thoughts of Tomorrow, Journal of the American Statistical Association, Vol. 95, No. 452, (Dec., 2000) pp. 1269-1276

Boyce, A. M. (1999) Risk Analysis for Privately Funded Transport Schemes, European Transport Conference Proceeding Paper

Boyce, A. M. and Bright, M.J. (2003) Reducing or managing the Forecasting Risk in Privately Financed Projects, European Transport Conference Proceeding Paper

Brett, A. and Snelson, P. (1999) Traffic and Revenue Forecasting for Privately Funded Transport Projects, European Transport Conference Proceeding Paper

Brooks, S. P. (1998) Markov Chain Monte Carlo Method and its Application, The Statistician 47, part 1, pp69-100

Cheung, K. (2008) Bayesian Analysis for Modelling Uncertainty In Toll Road Demand And Revenue Forecasts, Unpublished MSc Dissertation, Imperial College London

De Jong, G. de., Pieters, M., Miller, S., Daly, A., Plasmeijer, R., Graafland, I., Lierens, A., Baak, J., Walker, W., and Kroes, E. (2005) Uncertainty in Traffic Forecasts, AVV Transport Research Centre, Rand Europe

DERSA (2008) Rodoanel Mario Covas. [online] Available from: http://www.dersa.com.br/rodoanel/especial/ [Accessed 02-01-2009]

Evans S.P. (1976) Derivation and Analysis of Some Models for Combining Trip Distribution and Assignment", Transportation Research, 10, pp. 37-57.

Flyvbjerg, B., Skamris Holm, M. K., and Buhl, S. L. (2006) Inaccuracy in Traffic Forecasts, Transport Reviews, Vol. 26, No. 1, 1–24, January 2006

Gaudry, M. J. I., Jara Diaz. S. R., and Ortuzar. J. de D. (1989) Value of time sensitivity to model specification, Transportation Research B23, 151-158

Geman, S. and Geman, D. (1984) Stochastic relaxation, Gibbs distribution and Bayesian restoration of images. IEE Transactions on Pattern Analysis and Machine Intelligence 6: 721–741

Gilks, W. R., Richardson, S., and Spiegelhalter D. J. (1996) Markov Chain Monte Carlo in Practice, Chapman and Hall

Harris, C., Hodges, J., Schur, M., and Shukla, P. (2003) Infrastructure Projects: A Review of Cancelled Projects, Public Policy for the Private Sector Note Number 252, The World Bank

Hastings, W. K. (1970), Monte Carlo sampling methods using Markov chains and their applications, Biometrika, 57, 97–109.

Jensen, F. V. (1996) An Introduction to Bayesian Networks, UCL Press

Linveld. C., Logie. M., Polak. J. W. (2006) A Bayesian Framework for the Integration of Multiple Transport Information Sources, Journal of Transportation Research Record

Lunn, D.J., Thomas, A., Best, N., and Spiegelhalter, D. (2000) WinBUGS -- a Bayesian modelling framework: concepts, structure, and extensibility. Statistics and Computing, 10:325--337.

Metropolis, N., Rosenbluth, A. E., Rosenbluth, M. N., Teller, A. H., and Teller, E. (1953) Perspective on “Equation of state calculations by fast computing machines”, J Chem Phys 21: 1087±1092

Montironi, R., Whimster, W. F., Collan, Y., Hamilton, P. W., Thompson, D., and Bartels, P. H. (1996) How to develop and use a Bayesian Belief Network, Journal of Clinical Pathology 1996;49:194-201

Newell, F. G. (1971) Applications of Queuing Theory, Chapman and Hall

Palisade (2002) @Risk: Simulation Add-in for Microsoft Excel, Windows version release 4.5.2 Industrial Edition, Newfield N.Y., Palisade Corporation

Pourret, O., Naim, P., and Marcot, B. (2008) Bayesian Networks A Practical Guide to Applications, Wiley

Raiffa, H. (1968) Decision Analysis: Introductory Lectures on Choices Under Uncertainty. McGraw-Hill College Custom Series

Ross, S. R., Westerfield, R. W., Jordan, B. D. (2008) Corporate Finance Fundamentals, McGraw-Hill Irwin

Scollnik, D. P. M. (2001) Actuarial Modeling with MCMC and BUGS. North American Actuarial Journal, 5 (2), pages 96-124

Standard & Poor’s (2002) Infrastructure Finance Traffic Risk in Start-up Toll Facilities

Verhoeven, M., Arentze, T., Timmermans, H., and Waerden, P. v. d. (2004) Modeling the Impact of Key Events on Long-Term Transport Mode Choice Decisions: A Decision Network Approach Using Event History Data, Paper submitted for 84th Annual Meeting of Transportation Research Board, Washington

Wardrop, J. G. (1952) Some Theoretical Aspects of Road Traffic Research. Proceedings of the Institution of Civil Engineers, Part II, Volume I, pp. 325-362.

World Bank (2006) The World Bank PPI Database

Yescombe, E. R. (2002) Principles of Project Finance, Academic Press

Zhao, Y. and Kockelman, K. M. (2001) The Propagation of Uncertainty Through Travel Demand Models: An Exploratory Analysis, Annals of Regional Science

