A NOVEL DESIGN AND IMPLEMENTATION OF AN AGGREGATE UK-BASED TRANSPORT MODEL

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1. INTRODUCTION

This paper presents a flexible, aggregate transport modelling framework with state-of-the-art model design and implementation that takes advantage of recent advances in computer power to provide robust demand forecasts within a reasonable timeframe.

The Great Bristol Modelling Framework (GBMF) has been developed as a six-stage, fully WebTAG-compliant (DfT, 2004 onwards) aggregate transport model system to assess a range of potential transport interventions in the West of England. The GBMF system has been successfully applied to assess a range of transport interventions (including demand management) and has supported a number of a Major Scheme Bid (MSB) submissions seeking UK Central Government funding.

Within the UK, the majority of the transport modelling undertaken for large urban areas continues to use aggregate models, reflecting the extensive experience in developing and applying them coupled with the typically smaller data requirements. Data collection in the UK for transport appraisal typically involves roadside interview data (RSI), onboard public transport surveys, manual / automatic count data, and journey time data etc rather than much larger, and more expensive, household interview data and revealed / stated preference data required to support disaggregate choice models.

To support the practitioner in developing MSB submissions, the UK Department for Transport (DfT) provides their WebTAG guidance on the building of stage-based transport models including the option to use imported sensitivity parameters (referred to as 'illustrative' parameters in WebTAG) rather than estimated parameters derived from more local data sources. The suitability of the model for scheme appraisal is demonstrated through a series of realism tests to ensure that the model realistically represents locally observed behavioural characteristics.

The GBMF modelling system was developed using INRO's EMME multi-modal and SATURN (Van Vliet et al, 1980) highway assignment software suites and includes a number of innovative features such as:

- One of the first fully WebTAG-compliant demand models in the UK;
- A six-stage model representing a 24-hour travel demand in Production/Attraction (P/A) format using an incremental, logit-based pivot-point form to achieve equilibrium in forecasting mode;
- Fully multi-modal functionality with detailed representation of travel demand by purpose, mode, car availability and income group (within both the supply and demand-side sub-models) to asses demand management schemes;

- A new P/A-based time of day choice implementation, derived specifically for this study, that enables time period choice to be undertaken after main mode choice but before destination choice;

- The adoption of cost-dampening and the introduction of further segmentation by distance-based value of time to achieve the required outturn elasticities for longer-distance trips; and

- Recent developments in SATURN including both optimising the model for running on quad-core desktop computers and the use of SATURN-CASSINI software that enables the convergence targets in the supply-side model to be dynamically linked to those obtained within the demand model.

This rest of the paper is structured as follows: Section 2 presents an overview of the GBMF system whilst Section 3 describes the technical innovations developed for GBMF including the use of High Performance Computing (HPC) and SATURN-CASSINI to significantly reduce overall model runtimes along with a new PA-based time-of-day choice formulation. Section 4 presents a comparison of the calibrated model parameters and resulting outturn elasticities, before-and-after the use of a cost-dampening mechanism, which is applied through the introduction of Value of Time (VOT) variation with distance. A summary of the model development is provided in Section 5.

2. GBMF MODELLING SYSTEM

FULLY WEBTAG-COMPLIANT CHOICE STRUCTURE

The GBMF is an aggregate transport modelling framework consisting of six stages to represent average weekday traffic conditions. It consists of five EMME-based demand modelling stages plus a separate supply-side route choice assignment stage for highway (SATURN) and Public Transport (EMME), as detailed in Table 1.

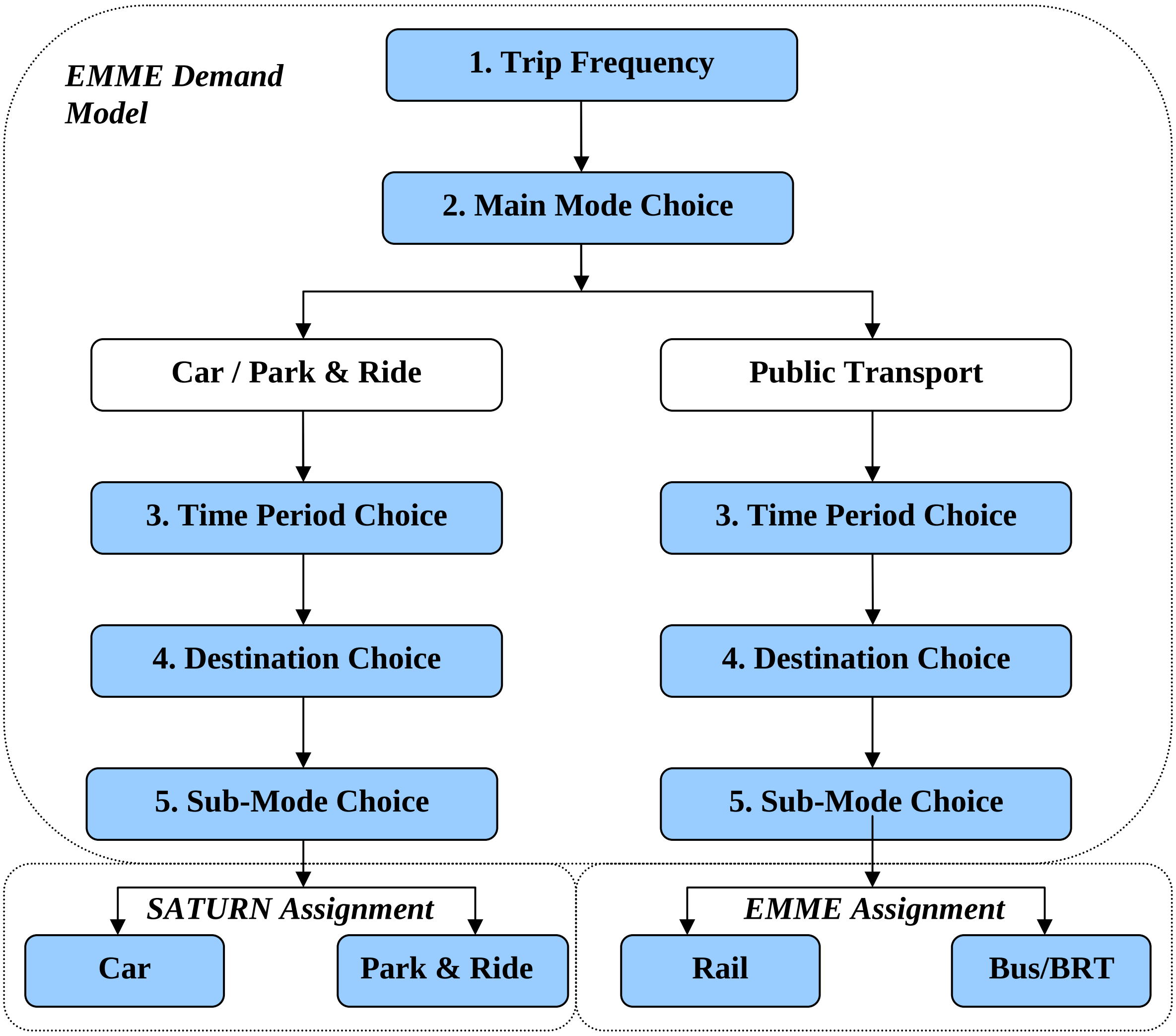
Table 1 - GBMF Demand Model Stages

Stage	Model	Temporal Scope	Form	Person Type
1	Frequency Modelling	24-hour	P/A Trip-ends	All (CA & NCA)
2	Main Mode Choice	24-hour	P/A Trip-ends	CA
3	Time Period Choice	Translate 24-hour to AM (3hr), IP (6hr), PM (3hr) and OP (12hr) periods	P/A Trip-ends	All (CA & NCA)
4	Destination Choice	3hr (AM), 6hr (IP), PM (3hr) and OP (12hr)	Translate P/A Trip-ends to P/A matrices	All (CA & NCA)
5	Sub Mode Choice	3hr (AM), 6hr (IP), PM (3hr) and OP (12hr)	P/A matrices	All (CA & NCA)
6	Assignment	1-hour (AM, IP, PM)	O/D matrices	All (CA & NCA)

Note: CA and NCA refer to person types with or without car available respectively whilst AM (07:00-10:00), IP (10:00-16:00), PM (16:00-19:00) and OP (19:00-07:00) represents the time period specification in a 24-hour weekday time frame. The conversion from P/A to O/D form is described in more detail in Section 3.

The GBMF demand model is an incremental logit-based model following the WebTAG guidance. Figure 1 shows the demand model hierarchical structure with the frequency modelling placed at the top and the sub mode choice for both cars and PT placed at the bottom.

Figure 1 - GBMF Demand Model Structure



Note that the non-motorised modes such as walk and cycles are not explicitly modelled in GBMF. There potential impact on demand changes is represented through the trip frequency sub-model as permitted by WebTAG.

Park and ride (P&R) is treated as a sub mode of car. P&R modelling uses an absolute logit model of site choice to enable P&R users to choose between competing sites within their catchment areas.

Level of Segmentation

Demand Segmentation (Stages 1 to 5)

To support a range of Major Scheme Bid submissions including the appraisal of road-user charging schemes, travel demand is disaggregated into sixteen segments as summarised below in Table 2. Travel demand is separated into five different journey purposes (home-based work, home-based other, non home-based other, home-based employers business and non home-based employers business), three household income bands (low, median and high) and two person types (car-available and non-car available). The level of segmentation is more detailed than the minimum suggested by WebTAG.

Table 2 – Demand Model Segmentation

Supply Purpose	Demand Purpose	Car Available (CA)			Non Car Available (NCA)
		Income Low	Income Medium	Income High	
Other	HBO	1	2	3	12
	NHBO	4	5	6	13
Work	NHBEB	7			14
	HBEB	8			15
Commuting	HBW	9	10	11	16

In addition, the GBMF separately represents light and heavy good vehicles but these are not considered within the demand model.

Supply-Side Segmentation (Stage 6)

For highway assignment modelling, the GBMF demand is aggregated, by income group, into four user classes within each time period plus a further two user classes for light and heavy goods vehicles as summarised in Table 3. The aggregation is undertaken by income group as the route choice coefficients for the assignment are the same and enables the overall CPU time to be significantly, reduced.

Table 3 – Highway Assignment User Classes (by Time Period)

User Class	Description	Demand Segments	Demand Responsive
1	Car Non Work Income Low	1, 4, 9	Yes
2	Car Non Work Income Median	2, 5, 10	Yes
3	Car Non Work Income High	3, 6, 11	Yes
4	Car Work	7, 8	Yes
5	Light Goods Vehicles	Not represented	No
6	Heavy Goods Vehicles	Not represented	No

A higher level of aggregation is undertaken in the public transport assignment model with segmentation only applied by mode (ie bus, rail, BRT/LRT) reflecting limitations within the EMME software.

The three (one-hour) supply-side models provide costs for the AM peak period (07:00-10:00), Inter-peak period (10:00-16:00) and PM peak period (16:00-19:00). Separate pre-peak hour models represent the travel conditions in the 07:00-08:00 and 16:00-17:00 time period for the AM and PM peak hours respectively. The demand for these pre-peak hours is a proportion of the peak hours with the factor derived from local traffic count profiles.

To reduce model runtimes, it is assumed that that the assignment costs for the Off-Peak (19:00-07:00) representing the travel conditions in that period are a proportion of the inter-peak period costs.

3. TECHNICAL CHALLENGES & INNOVATIONS

HIGH PERFORMANCE COMPUTING

Convergence Targets

The GBMF demand model, highway and public transport assignment models are linked together with demand and supply loops controlled in an automated fashion. The traditional Cobweb averaging method was implemented to determine the equilibrium point between the demand and supply models and the %GAP measure (as defined in WebTAG Unit 3.10.4) was used to monitor the level of convergence achieved. The %GAP measure (across all segments) is defined as:

$$\frac{\sum_{ijctm} C\left(X_{ijctm}\right)\left|D\left(C\left(X_{ijctm}\right)\right)-X_{ijctm}\right|}{\sum_{ijctm} C\left(X_{ijctm}\right)X_{ijctm}} * 100$$

where:

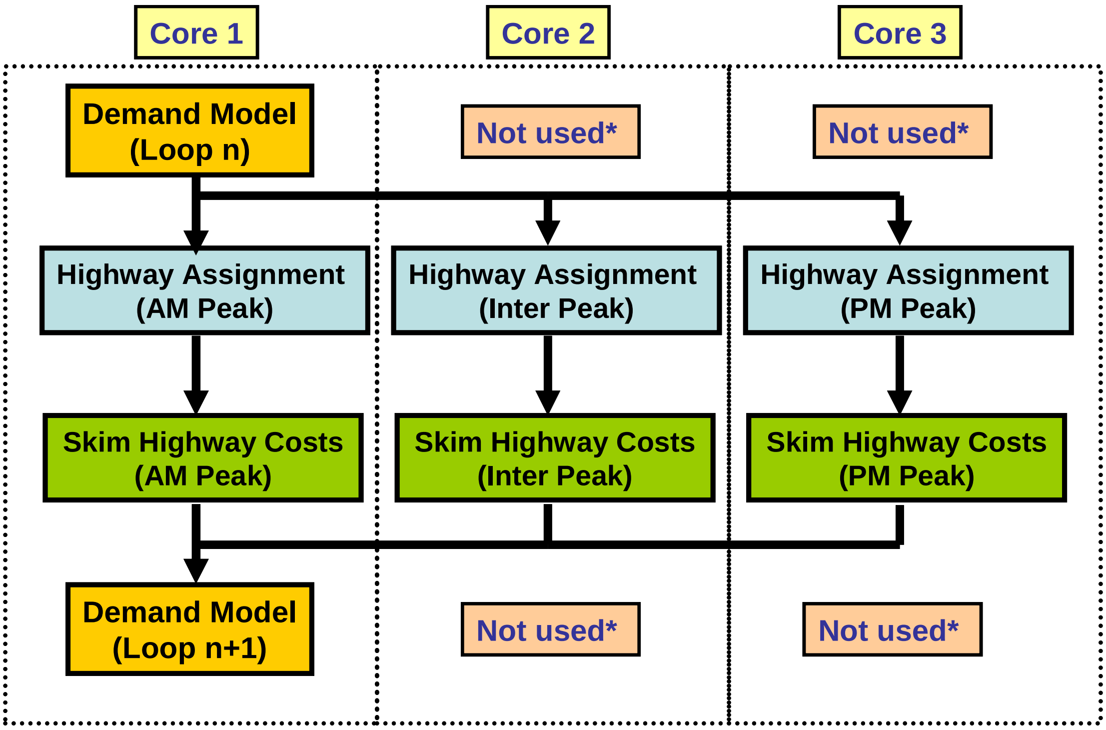
- X_{ijctm} is the current flow vector or matrix from the model
- $C(X_{ijctm})$ is the generalised cost vector or matrix obtained by assigning that matrix
- $D(C(X_{ijctm}))$ is the flow vector or matrix output by the demand model, using the costs $C(X_{ijctm})$ as input; and
- $_{ijctm}$ represents origin *i*, destination *j*, demand segment/user class *c*, time period *t* and mode *m*.

The GBMF model is run until the convergence level, as measured by the aforementioned %GAP, is lower than 0.2%. Experimentation has shown that this requires the highway convergence levels as defined by Wardrop's equilibrium to be less than 0.05%. This is a very high level of convergence and incurs a substantial CPU overhead.

Model Runtimes

The largest of the GBMF models has 600 zones and on a standard high-spec Pentium-4 based desktop PC required more than 100+ hours of CPU time to achieve the required convergence target. The majority of the CPU time was required to undertake the highway assignments (and subsequent skimming). To reduce model runtimes, a number of changes were made including the use of the latest Intel Quad-core Xeon (X5450) workstations, simultaneous skimming of time, distance and tolls using the new SKIM_ALL option in SATURN and the parallelization of the assignment and skimming by time period across the individual CPU cores as illustrated below in Figure 2.

Figure 1 - Parallel SATURN Highway Procedure



***Not used by SATURN**

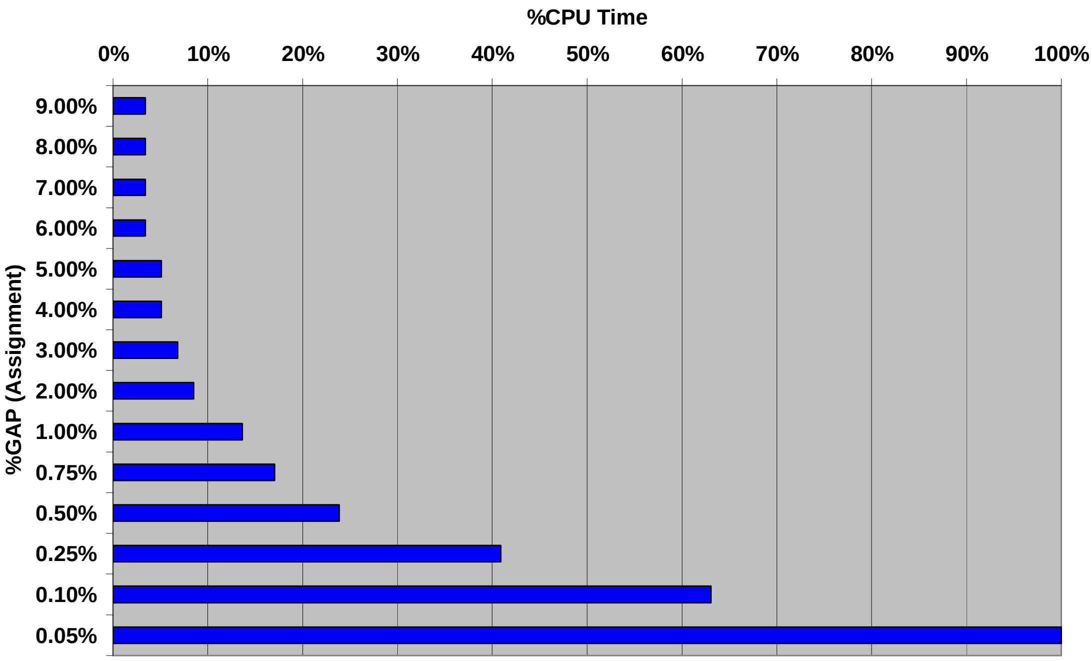
The control of the parallel process was undertaken by the new SATURN MONITOR and WAIT programs called from within the EMME demand model.

VARIABLE CONVERGENCE TARGETS (SATURN CASSINI)

Highway Convergence

The SATURN assignment algorithm uses the Frank-Wolfe algorithm (Frank and Wolfe, 1956) to achieve an equilibrium solution. A characteristic of the algorithm is a rapid initial decent before a gradual approach to a highly converged solution as shown in Figure 3. For example, to achieve a %GAP value of 0.05 requires around 20 times the CPU time to achieve a %GAP of 5.0, eight times the time to achieve a %GAP of 1.0 and four times the time to achieve a %GAP of 0.5. Clearly, significant CPU savings may be achieved by (appropriately) reducing the convergence targets for the SATURN highway model where possible.

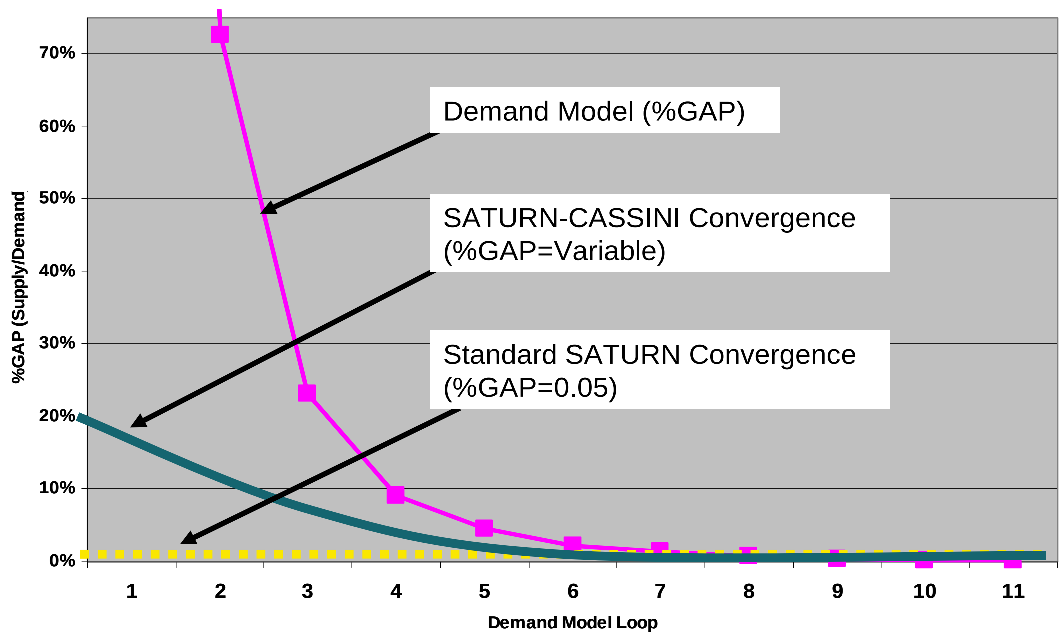
Figure 3 – SATURN Assignment: %CPU time to converge to %GAP=0.05



Demand Model Convergence

As noted earlier, the demand model uses a Cobweb method to achieve convergence between the supply and demand models. The convergence profile of the demand model is similar to the highway model as shown below in Figure 4. By setting a more relaxed highway convergence target for the early demand model loops, considerable savings in CPU time may be achieved. This is undertaken automatically by the new SATURN-CASSINI (Atkins, 2009) program that tracks the level of convergence achieved within the Demand Model and sets appropriate targets for the highway model. The two profiles are also shown in Figure 4.

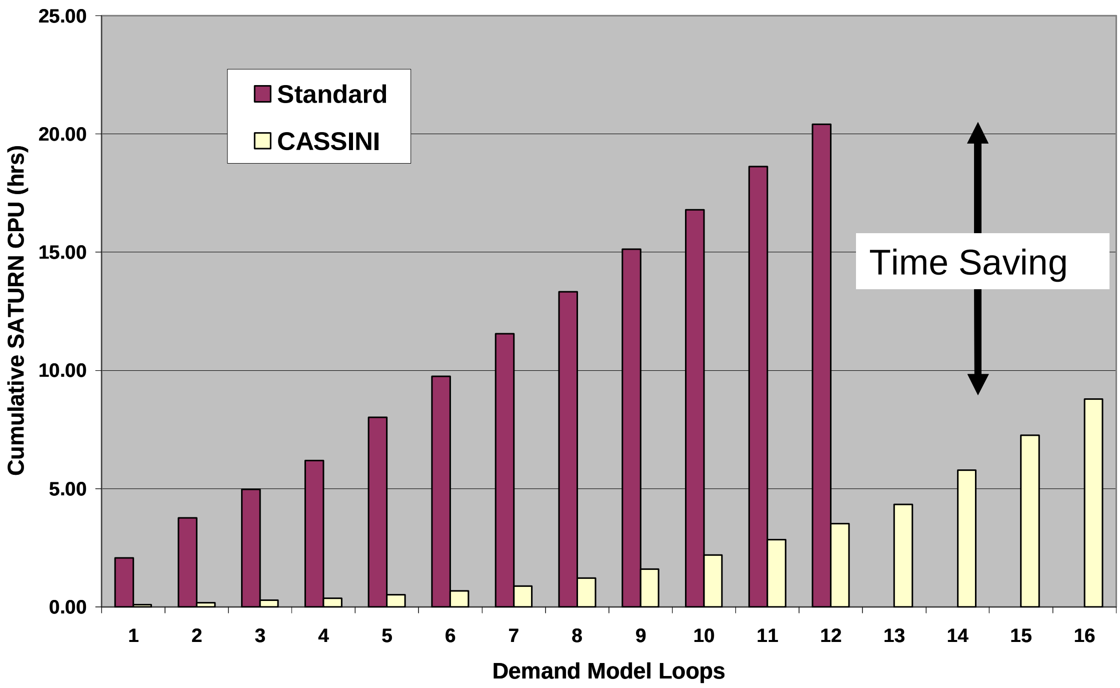
Figure 4 – Variable Convergence Targets



SATURN-CASSINI

With the SATURN-CASSINI program, a considerable reduction in CPU time is achieved for the early loops of the demand model. Whilst the demand model requires more loops to achieve the same %GAP value of <0.2 – typically an extra three or four loops reflecting the slower descent – there is an overall saving of around 40% in the highway CPU time compared to the standard method as shown in Figure 5 below.

Figure 5 – Comparison of GBMF Highway Model Runtimes by Demand Loop



With these innovations in optimising the CPU usage, CASSINI has reduced model runtimes from around 30+ hours to around 18 hours per 2031 forecast year scenario.

The recent introduction of SATURN Multi-Core, the new multi-threaded version of the SATURN assignment program, has further reduced overall CPU times to less than 13 hours – a further reduction of over 25%.

PRODUCTION-ATTRACTIONS WITH TIME PERIOD CHOICE

The introduction of PA-based modelling, as required by WebTAG, with the explicit consideration of time period choice is complex particularly when, as shown previously in Figure 1, time period choice is undertaken after main mode choice but before destination choice. The key technical challenge, within the demand model, was how the demand and costs arising from the return leg of a home-based trip may be estimated when the timing of the return leg is dependent on the earlier outward journey.

In other words, if an outward home-based trip retimed from the morning peak period to the corresponding inter-peak period in response to the introduction of a morning peak road pricing, when would the corresponding return leg be undertaken?

Within the demand model, the challenge was to determine the appropriate travel demand and associated costs of return-legs of home based trips in a coherent and consistent manner given that the return-leg journeys were constrained by the nature of their outward journeys. Whilst WebTAG requires that a PA functional form should be adopted, it did not provide any guidance on how it may be implemented.

Methodology

A new PA-based formulation with time period choice was proposed. The fundamental assumptions underpinning this approach are the use of fixed return proportions and time of day restriction:

- For outward trips leaving home within each time period, the proportions of trips returning in subsequently time periods remain fixed by purpose over the base year and future forecast years. In other words, if AM tolls were applied and certain trips shifted to the IP period (for example), the return leg of these transferred outward trips would have the same return patterns as those already established in the base inter-peak period;

- The time of a day choice starts with the AM Peak period and that trips departing over the course of the day will all return before the commencement of the following AM Peak period the next day. In other words, for each outbound from-home trip, there would be an equivalent trip returning home during the day and the sum of outward journeys equals to the sum of return journeys.

Accordingly therefore, only the outward from-home trips in each time period are explicit modelling variables for the PA formulation within the GBMF demand model. The return-leg demands were calculated from outward-leg demands with associated return proportions. The following paragraphs describe the details of the PA formulation when the time period choice is structured as previously shown in Figure 1.

Time Periods

Denote the modelled time period as (t), outward from-home time period as (*s*), and return to-home time period as (*r*) respectively. The four time periods (*t*) in a 24-hour day are:

t=am: 07:00-10:00; t=ip: 10:00-16:00; t=pm: 16:00-19:00; and t=op: 19:00-07:00

For a given time period *t*, the outward from-home time period (*s*) is the same as *t*:

$s = t \text{ for } t \in \{am, ip, pm, op\}.$

For each time period t (or s), there are multiple corresponding return time periods (r) as defined below:

$r \in \{am, ip, pm, op\}$, *if t = am;*

$r \in \{ip, pm, op\}$, *if t = ip;*

$r \in \{pm, op\}$, *if t = pm; and*

$r \in \{op\}$ *if t = op.*

The above relationship is illustrated below in Table 2 where symbol (√) indicates available returning time periods for each outward time period:

Table 2 - Returning Time Period Specification

		Return To-Home time period (r)			
		AM	**IP**	**PM**	**OP**
Outward From-Home Period (s)	**AM**	√	√	√	√
	IP		√	√	√
	PM			√	√
	OP				√

Demand Model Variable Notations

We use "p.c.m" or "pcm" to represent segmentation used in the GBMF with combination of purpose (p), person type (c) (household income band and CA/NCA), and mode (m). Table 4 below provides the variables used for the PA specification.

Table 4 – Notation Used in the PA Formulation

Notation	Description	Source Data
$Pout^{0}_{IJpcmt}$	Given time period t, reference outward from-home trip proportion by p.c.m for origin sector I and destination sector J. These factors are used only once in creating base PA trips.	RSI data
$\Pr et^{0}_{pcmsr}$	Given time period s, fixed to-home proportion for trips returned in time period r by p.c.m. These factors are only segmented by p.c.m – not enough data is available to populate all ij pairs in a matrix from. (Not sure at this stage if sector based factors are achievable).	RSI data and NTS data
$T^{(RSI)}_{IJpcms}$	The total of from-home trips from 2006 RSI by p.c.m in time period s from origin sector I to destination sector J (directional from-home).	RSI data
$T^{(RSI)}_{IJpcmt}$	The total of from-home and to-home trips from RSI by *p.c.m* in time period *t* from origin sector *i* to destination sector *j* (non directional).	RSI data

Notation	Description	Source Data
$T^{(OD)0}_{ijpcmt}$	Reference OD assignment matrices from origin *i* to destination j in time period *t* by *p.c.m* (non directional).	Calibrated / Validated base assignment matrices
$T^{(OD)0}_{ijpcms}$	Reference outward OD trips from origin *i* to destination *j* in time period *s* by *p.c.m* (directional from-home).	
$T^{(PA)0}_{ijpcmt}$	Reference production-attraction (PA) trips from production zone i to attraction zone *j* in time period *t* by *p.c.m*.	
$C^{(PA)0}_{ijpcmt}$	Reference production-attraction (PA) costs from production zone *i* to attraction zone *j* in time period t by *p.c.m*.	
$C^{(OD)0}_{ijpcms}$	Skimmed base OD generalised costs of travel for outward trips in time period s from origin *i* to destination *j* by *p.c.m* (directional from-home).	
$C^{(OD)0}_{ijpcmtr}$	Given time period *t*, skimmed base OD generalised costs of travel for trips returning home in time period r from origin *i* to destination *j* by *p.c.m* (directional to-home)	
$T^{(PA)0}_{ijpcm24}$	Reference 24hr PA trips from production zone *i* to attraction zone *j* by *p.c.m.*	Fixed
$C^{(PA)}_{ijpcmt}$	PA costs of travel for time period *t* converted from relevant OD outward and return costs from production zone *i* to attraction zone *j* by *p.c.m.*	
$C^{(OD)}_{ijpcms}$	Skimmed OD generalised costs of travel for outward trips in time period s from origin *i* to destination *j* by *p.c.m* (directional from-home).	
$C^{(OD)}_{ijpcmtr}$	Given time period *t*, skimmed OD generalised costs of travel for trips returning home in time period r from origin *i* to destination *j* by *p.c.m* (directional to-home)	
$\Delta C^{(PA)}_{ijpcmt}$	The change of PA costs from the forecast year over the base year from production zone *i* to attraction zone *j* in time period *t* by *p.c.m.*	WebTAG
CC	Composite costs (logsums) over IHL	WebTAG
λ	A series of IHL Spreading parameters over FMTD stages	Subject to realism tests
$T^{(PA)}_{ijpcmt}$	Latest production-attraction (PA) trips from production zone *i* to attraction zone *j* in time period *t* by *p.c.m*.	Output directly from the demand model
$T^{(OD)}_{ijpcms}$	Estimated OD outward trips from origin *i* to destination *j* in time period *s* by *p.c.m* (directional from-home).	
$T^{(OD)}_{ijpcmtr}$	Given time period *t*, estimated OD return trips that happen in time period *r* from origin *i* to destination *j* by *p.c.m* (directional to-home).	
$T^{(OD)}_{ijpcmt}$	Given time period *t*, the latest total OD trips estimated in the current demand/supply loop from origin *i* to destination *j* in time period *t* by *p.c.m* (non directional).	Send to the assignment stage

Create Outward and Return Proportions

For a given time period *t*, the reference proportion of outward (from-home) trips over total trips was calculated via RSI data, which should only be used once to create reference PA matrices by time period and by segmentation:

$$Pout^{0}_{IJpcmt} = \frac{T^{(RSI)}_{IJpcms}}{T^{(RSI)}_{IJpcmt}} \qquad (1)$$

The fixed reference proportions by time period *s* for GBMF were based on National Travel Survey (NTS) data supplied by DfT (as shown below in Table 5). Note that for a given time period *s*, proportions for trips returning home in time period *r* are subject to the following constraint:

$$\sum_{r} \mathrm{Pret}^{0}_{pcmsr} = 1 \qquad (2)$$

Table 3: Example Set of Fixed Return Proportions

	HBW	HBO	HBEB
AM Outward			
AM Return	0.03	0.26	0.06
IP Return	0.20	0.55	0.55
PM Return	0.67	0.15	0.31
OP Return	0.10	0.04	0.08
Total	***1.00***	***1.00***	***1.00***
IP Outward			
IP Return	0.26	0.70	0.81
PM Return	0.49	0.25	0.16
OP Return	0.25	0.05	0.03
Total	***1.00***	***1.00***	***1.00***
PM Outward			
PM Return	0.48	0.58	0.40
OP Return	0.52	0.42	0.60
Total	***1.00***	***1.00***	***1.00***
OP Outward			
OP Return	1.00	1.00	1.00
Total	***1.00***	***1.00***	***1.00***

Create Reference PA Costs and Demands

For a given time period *t*, reference demands and costs were calculated by the following two formula respectively:

$$T^{(PA)0}_{ijpcmt} = T^{(OD)0}_{ijpcms} = T^{(OD)0}_{ijpcmt} Pout^{0}_{ijpcmt} \quad (3)$$

$$C^{(PA)0}_{ijpcmt} = (C^{(OD)0}_{ijpcms} + \sum_{r \geq s} (C^{(OD)0}_{ijpcmtr})' \Pr et^{0}_{pcmsr})/2 \quad (4)$$

where $r \geq s$ means that r ranges from the outward from-home time period (*s*) up to the last time period (*op*) in a day, and the (·)' means a transpose of a matrix. In other words, the costs defined in (4) are a weighted average of the outward and return legs.

The daily 24-hour reference demand was the sum of the time period PA demands (which account for only an half of total OD demands):

$$T^{(PA)0}_{ijpcm24} = \sum_{t} T^{(PA)0}_{ijpcmt} \quad (5)$$

Convert OD Costs to PA Costs

For each demand / supply loop, the skims from the OD-based assignment by time period (*t*) were converted to PA costs for feeding into the demand model. With the same formulation as given by (4), the PA costs in forecasting considered both outward and return journeys simultaneously as a weighted sum given below:

$$C^{(PA)}_{ijpcmt} = (C^{(OD)}_{ijpcms} + \sum_{r \geq s} (C^{(OD)}_{ijpcmtr})' \Pr et^{0}_{pcmsr})/2, \quad (6)$$

where $r \geq s$ means that *r* ranges from the outward from-home time period (*s*) up to the last time period (*op*) in a day.

By adding the relevant return costs, say, any AM tolls will be appropriately allocated to further to-home trips occurring in the same and subsequent time periods (i.e. IP, PM and OP), and therefore the impact of AM tolls will be distributed across all time periods rather than incorrectly allocated to the AM demand calculation only.

Incremental Demand Modelling

For an Incremental Hierarchical Logit (IHL) formulation as used in GBMF, the change of PA costs at the bottom level of hierarchy over the base year was simply defined as

$$\Delta C^{(PA)}_{ijpcmt} = C^{(PA)}_{ijpcmt} - C^{(PA)0}_{ijpcmt} \quad (7)$$

Based on $\Delta C^{(PA)}_{ijpcmt}$, the composite costs i.e. the structured logsums over the various stages of the demand model were calculated in the standard way:

$$CC = f(\Delta C^{(PA)}_{ijpcmt}, T^{(PA)0}_{ijpcmt}, \lambda) \quad (8)$$

Based on the CC and others, the demand model calculated a new set of PA outward-leg demands for each demand/supply loop, or simply

$$T_{ijpcmt}^{(PA)} = f(CC, T_{ijpcmt}^{(PA)0}, \lambda)$$

Convert PA Demands to OD for Assignment

The outward PA demands $T_{ijpcmt}^{(PA)}$ output from the demand model were then converted to the OD form for assignment. The outward from-home OD demands are simply the latest PA demands output from the demand model:

$$T_{ijpcms}^{(OD)} = T_{ijpcmt}^{(PA)} \qquad (9)$$

Return-leg demands were constrained by relevant outward from-home trips that take place in previous time periods. Note that the PM return demands corresponded to proportions of trips travelling out in the AM period, IP period, and PM period respectively.

For given time period (t), the formula to calculate to-home demands is given below by applying the fixed return proportions over the latest outward from-home trips:

$$T_{ijpcmtr}^{(OD)} = \sum_{s \le r} \mathrm{Pr}\, et_{pcmsr}^{0} (T_{ijpcms}^{(OD)})', \qquad (10)$$

where $s \le r$ means that s ranges from the first time period (AM) up to the current time period t.

Finally, the OD assignment demands were simply the sum of from-home and to-home trips:

$$T_{ijpcmt}^{(OD)} = T_{ijpcms}^{(OD)} + T_{ijpcmtr}^{(OD)} \qquad (11)$$

Final Comments

The demand model estimates the outward PA demands directly by standard IHL technique. The return-leg demands were implicitly considered via the outward journeys in the following way:

- Return OD costs were incorporated in formulas (4) and (6) above, i.e. the PA costs are taken as the average OD costs between the outward and return journeys;

- Return-leg trips were collected by formula (10) from their relevant outward legs using fixed return proportions. Therefore, any reduction of AM trips resulting from say, the introduction of AM tolls, would have been mapped onto the corresponding return legs.

COST DAMPENING: VALUE OF TIME VARIATION WITH DISTANCE

Overview

An inherent property of incremental hierarchic logit models is the difficulty in achieving observed outturn elasticities for both shorter and longer distance trips. The calibration of the initial GBMF (referred to as 'version 1 model' herein) focussed on achieving the appropriate outturn elasticities for shorter-distance trips, reflecting the main area of interest for the study. However, as subsequently described in Section 4, the resulting outturn elasticities were notably higher than the target elasticities as recommended by WebTAG.

Variations in Value of Time

To overcome this problem, a form of cost dampening was introduced (as now outlined in the latest WebTAG Unit 3.10.4) whereby the Value of Time (VOT) varied with distance for non-work trips. The adoption of VOT variation in the subsequent version 2 enabled the model to successfully replicate the target elasticities.

WebTAG Unit 3.12.2 (para 11.4.2) provides a formula to estimate local VOTs for modelling road pricing schemes if the full distribution of income and distance of trips is known. For GBMF, the average household income for all movements was incomplete so an alternative expression of the WebTAG formula was derived as presented below:

$$VOT = \max\left(VOT_C \left[\frac{D}{D_0} \right]^{\eta_s}, VOT_C \left[\frac{D_C}{D_0} \right]^{\eta_s} \right)$$

where:

- *VOT* : value of time used in version 2 for non-work trips (which varies by distance);
- VOT_C : the central value of time derived from WebTAG for non-work trips;
- D: length of trip (as estimated below); and
- D_0, D_C, and η_s: parameters.

The formula provides a matrix of VOTs by distance for non-work trips with the trip length D representing the distance between each i-j pair in typical free-flowing conditions. The distance elasticity parameter (η_s) is provided in para 11.4.4 of WebTAG Unit 3.12.2 and is set to: 0.314 for other trips (HBO / NHBO) and 0.421 for commuting trips (HBW), together with the distance parameter D_0 as 7.58 miles (12.2 kilometres). The value of D_C was set to 4km to identify the very short distance trips (including intra-zonal movements).

Table 6 below summaries the VOTs used in both GBMF v1 and v2 models by income group and purpose – the VOT variation by distance used in the v2 model is represented by the matrix average, minimum and maximum. Note that there was no variation in VOTs for either car-based Home-Based Work (as the outturn elasticities were already within the acceptable range defined in WebTAG) or work trips (single income group).

Table 6: Variation in Value of Time with Distance

Purpose	Car Available (HBO / NHBO)			Non Car Available	
	Income Low	Income Median	Income High	HBW	HBO / NHBO
GBMF Version 1 (Without VoT Variation)					
Central Value	7.08	9.14	10.98	6.66	5.49
GBMF Version 2 (With VoT Variation)					
Average Value	8.40	10.84	13.02	8.57	6.51
Minimum Value	4.99	6.44	7.74	4.16	3.87
Maximum Value	26.80	34.60	51.57	39.69	20.78

All values in pence / minute.

4. MODEL CALIBRATION AND OUTTURN ELASTICITIES

MODEL CALIBRATION

WebTAG provides a range of illustrative model parameters for use in models where locally observed data is not available. WebTAG recommends that model parameters should be within this range (as outlined in WebTAG 3.10.3), as they are derived from a number of UK models developed using locally collected Revealed / Stated Preference survey datasets.

The following paragraphs compare the model sensitivity parameters and outturn elasticities for the GBMF version 1 and version 2 implementations (ie before and after the introduction of VOT variation with distance)

COMPARISON OF MODEL SENSITIVITY PARAMETERS

Table 7 compares the model scale parameters (Theta) between lower level destination choice lambdas and upper level mode choice lambdas for version 1 and version 2 models alongside the range of illustrative WebTAG parameters for each purpose. Note that time period choice scale parameters used the same values as the main mode choice in GBMF as recommended by WebTAG and are, in effect, undertaken simultaneously.

Table 7 shows that the introduction of variation in VOT by distance enabled the calibrated Theta parameters for the version 2 model to match the median WebTAG values. Conversely, several of the version 1 model values were significantly below the minimum recommended WebTAG target values as indicated by (*).

Table 7 – Comparison of Scale Parameters in Mode Choice

Purpose	WebTAG Theta (Minimum / Median / Maximum)	GBMF Theta
GBMF Version 1 (Without VOT Variation)		
HBO	0.27 / 0.53 / 1.00	0.51
NHBO	0.62 / 0.81 / 1.00	0.48(*)
NHBEB	0.73 / 0.73 / 0.73	0.46(*)
HBEB	0.26 / 0.45 / 0.65	0.55
HBW	0.50 / 0.68 / 0.83	0.54
GBMF Version 2 (With VOT Variation)		
HBO	0.27 / 0.53 / 1.00	0.53
NHBO	0.62 / 0.81 / 1.00	0.81
NHBEB	0.73 / 0.73 / 0.73	0.73
HBEB	0.26 / 0.45 / 0.65	0.45
HBW	0.50 / 0.68 / 0.83	0.68

Table 8 presents the comparison of model sensitivity parameters (Lambda) for destination choice by mode (ie car versus public transport). As with the mode choice parameters, varying VOT by distance enables the version 2 model parameters to be set to the median WebTAG illustrative values for both modes whereas a number of the highway lambda values for version 1 model were below the minimum target values. Note that for both models, the public transport destination choice parameters were set to the median WebTAG values.

Table 8 - Comparison of Sensitivity Parameters in Destination Choice

Purpose	WebTAG Lambda		GBMF Lambda	
	Highway (Min / Median / Max)	Public Transport (Median)	Highway	Public Transport (CA / NCA)
GBMF Version 1 (Without VOT Variation)				
HBO	-0.074 / -0.090 / -0.160	-0.036	-0.070(*)	-0.036
NHBO	-0.073 / -0.077 / -0.105	-0.033	-0.069(*)	-0.033
NHBEB	-0.069 / -0.081/ -0.107	-0.042	-0.068(*)	-0.042
HBEB	-0.038 / -0.067 /-0.106	-0.036	-0.042	-0.036
HBW	-0.054 / -0.065 / -0.113	-0.033	-0.050(*)	-0.033
GBMF Version 2 (With VOT Variation)				
HBO	-0.074 / -0.090 / -0.160	-0.036	-0.090	-0.036
NHBO	-0.073 / -0.077 / -0.105	-0.033	-0.077	-0.033
NHBEB	-0.069 / -0.081 / -0.107	-0.042	-0.081	-0.042
HBEB	-0.038 / -0.067 / -0.106	-0.036	-0.067	-0.036
HBW	-0.054 / -0.065 / -0.113	-0.033	-0.085	-0.033

OUTTURN ELASTICITIES

WebTAG Target Values

WebTAG suggests that the fuel cost elasticities should be within the range of -0.1 to -0.4, with the more discretional trips (ie HBO / NHBO) closer to -0.4 whilst more compulsory trips such as Work (ie HBEB / NHBEB) close to -0.1. WebTAG also recommends that overall, all-day elasticity should be within the range of around -0.25 to -0.3, depending on the geographical location of the modelled are and the relative affluence to the UK as a whole. Using the aforementioned model parameters (ie lambda and theta values), outturn fuel-cost elasticities were estimated for the two GBMF model formulations.

Model Performance

Table 9 presents the outturn fuel cost elasticities for the 'Internal to Internal/External ('*I to I&E*') movements by time period and purpose where *I* represents the main modelled area and *E* represents the area outside. Note that in GBMF, external to external movements (*E to E)* were excluded from the demand model as the costs of those movements were not fully represented and were only subject to external growth factors (as were the light and heavy good vehicle movements).

Table 9 - Comparison of Outturn Fuel Elasticities

Time Period	Movement Type	HBW	HBO	NHBO	*Total Non-Work*	*Work*	*All*
GBMF Version 1 (Without VOT Variation)							
AM	I to I&E	-0.21	-0.85	-1.09	-0.42	-0.09	**-0.38**
IP	I to I&E	-0.16	-0.53	-1.01	-0.53	-0.07	**-0.43**
PM	I to I&E	-0.11	-0.45	-0.97	-0.35	-0.04	**-0.33**
All Day	I to I&E	-0.16	-0.59	-1.02	**-0.46**	**-0.07**	**-0.39**
GBMF Version 2 (With VOT Variation)							
AM	I to I&E	-0.29	-0.38	-0.39	-0.32	-0.12	**-0.30**
IP	I to I&E	-0.23	-0.27	-0.42	-0.29	-0.10	**-0.26**
PM	I to I&E	-0.14	-0.22	-0.42	-0.23	-0.06	**-0.22**
All Day	**I to I&E**	-0.23	-0.28	-0.41	**-0.28**	**-0.10**	**-0.26**

Table 9 shows that for the version 1 model (without VOT variation with distance), the outturn fuel cost elasticities were too high for the discretionary trips (HBO and NHBO) which, in turn, result in overall outturn all-day elasticities noticeably greater than the WebTAG target values. With the version 2 model, the introduction of VOT variation with distance enabled the outturn elasticities to fall within the WebTAG recommended range. The overall elasticity of -0.26 was lower than the maximum -0.30 target value reflecting the high income levels in the Greater Bristol area compared to the UK average.

5. SUMMARY

This paper presents a flexible, state-of-the-art aggregate transport modelling framework with innovative design and implementation. The GBMF framework represents the latest in UK best-practice for equilibrium-based demand models. GBMF includes a number of innovative features including:

- One of the first, fully-WebTAG compliant six -stage demand model representing a 24-hour travel demand in Production/Attraction (P/A) format using an incremental, logit-based pivot-point form to achieve equilibrium in forecasting;

- An innovative Production-Attraction (PA) formulation has been constructed for the GBMF with pseudo tours for average week days to enable the latest WebTAG requirements to be satisfied. Pseudo tours have been created based on trip data for home-based trips by using return proportions obtained from UK Department for Transport (DfT). This PA formulation is consistent to the GBMF demand modelling structure with pseudo tour zonal costs appropriately weighted between outward and return legs;

- The introduction of a form of cost dampening by distance-based value of time to achieve the required outturn elasticities for the fully represented movements within the study area; and

- The use of SATURN-CASSINI software - enabling the convergence targets in the supply-side model to be dynamically linked to those obtained within the demand model – has substantially reduced model runtimes. The recent migration to SATURN Multi-Core has provided further reductions to support the practitioner.

The GBMF has been used to support a number of Major Scheme Bid submission for UK Central Government funding and provides a modular framework that has been readily adapted to create a family of models of the Greater Bristol area.

6. REFERENCES

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